Engineer On a Disk

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email: jackh@gvsu.edu
phone: (616) 771-6755
fax: (616) 336-7215

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Electric Circuits

2. BASIC CIRCUIT ANALYSIS

• Circuit analysis can be troubling because we are dealing with particles that have never been seen. We depend upon calculations and approximations to determine what is happening inside the circuit, and measuring instruments to verify our numbers.

2.1 CIRCUIT COMPONENTS AND QUANTITIES

• Although in reality circuits involve complex interactions of potential and magnetic fields, we tend to simplify components into discrete and independant parts.

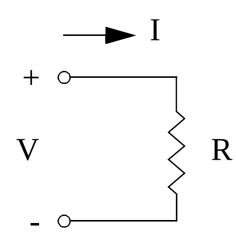
• Typically each simple circuit component will act as a “black box” with an applied current creating a voltage, or an applied voltage creating a current.

• Current and voltage are very important terms that are not well understood by the beginner. Consider an electron/proton pair. If both are together they are stable and steady. If we separate them they exert a force of attraction, much like gravity. This potential of attraction is called voltage. If we create a channel for these electrons to flow back to the protons (electrons are much lighter and more mobile than protons), the flow of electrons is called a current. The electrons do not flow freely, the restiction on flow is called resistance.

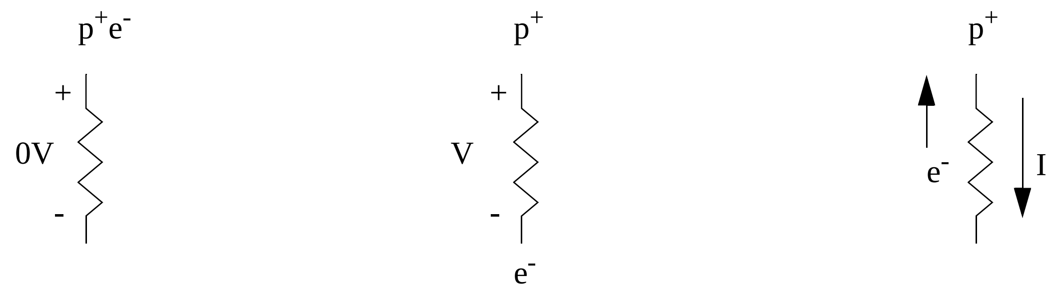
$$V = IR$$

where,

R = resistance (ohms Ω)
I = current (amperes, amps, A)
V = potential (voltage, V)



ASIDE: The electron is defined as a negative charge and the protons are positive charges. Because electrons are much lighter and smaller, they tend to move towards the positive charge. This gives the conductor the counter-intuitive result that while current is defined as flowing from positive to negative, the electrons are moving the opposite direction from negative to positive.



NOTE: Voltage is defined as a unit of energy per unit charge, such that,

$$v = \frac{dW}{dq}$$

where,
W = energy (joules)
q = the charge (coulombs)
V = the voltage

Current is defined as the unit of charge per unit of time,

$$I = \frac{dq}{dt}$$

Next consider the product of the current and voltage,

$$vI = \left(\frac{dW}{dq}\right)\left(\frac{dq}{dt}\right) = \frac{dW}{dt} = P$$

where,
P = the power in Watt's (joules/sec)

This gives the change in energy as a function of time, or the power.
And, if we consider resistance.

$$R = \frac{V}{I} = \frac{\left(\frac{dW}{dq}\right)}{\left(\frac{dq}{dt}\right)} = \frac{dWdt}{dq^2}$$

where,
R = resistance (Ω or Ws/C^2)

- Resistance is the simplest of all circuit elements, and is found in all circuit elements, but there are a variety of other simple elements found in circuits,
  - capacitors

- inductors
- voltage sources
- current sources

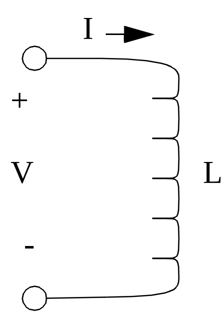
ASIDE: Resistance is caused by the current electrons moving through a conductor striking atoms and transferring energy to their electrons. (This causes the electrons on the effected atom to absorb momentum. The result is that there is an impeded (resisted) current flow, generating heat in the material.

• All of us have seen an electromagnet at least once in our lives. This is effectively a large inductor. The device is best described as resisting current flow changes (almost as if preserving the momentum of the current). The resulting relationship is,

$$V = L\frac{dI}{dt}$$

where,
V = the voltage
I = the current
L = the inductance (Henry's, H)

This may make more sense if we keep in mind that an inductor is just a coil of wire.



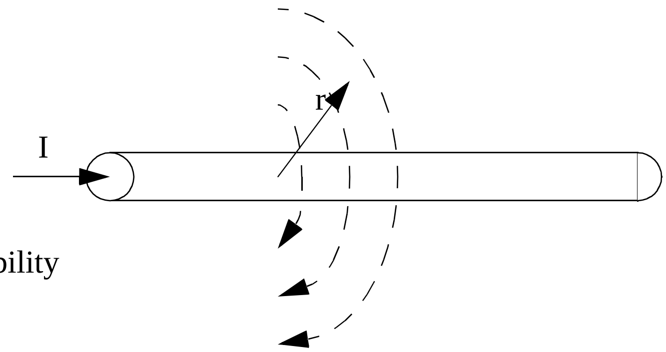
ASIDE: Inductors are based on the magnetic field created by a flow of charge (ie, a current). If the flow is constant a constant magnetic field builds up around the conductor. The strength of the field is given by,

$$H = \frac{I}{2\pi r}$$

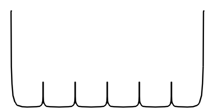
$$B = \mu H$$

where,

μ = magnetic permeability



The inductance of a straight conductor is small. In practical devices the inductors are wound into coils to increase the inductance. In a simple design the inductance is,



where,

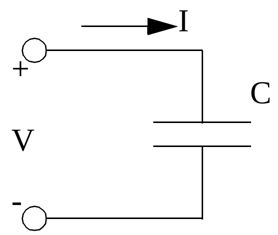
• When you get a static shock you are touching a basic form of capacitor. An electrical capacitor typically allows current to flow freely when a voltage is applied, but the current will quickly reach a steady stae. The relationship is,

$$I = C\frac{dV}{dt}$$

where,

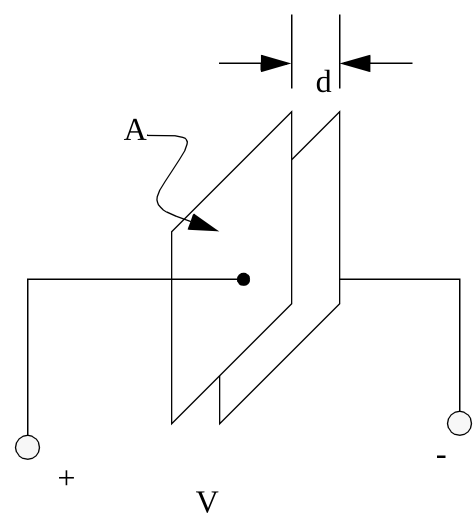
I = the current through the capacitor (A)
C = the capacitance (Farads, F)
V = the voltage across the capacitor (V)

The schematic symbol is shown below,



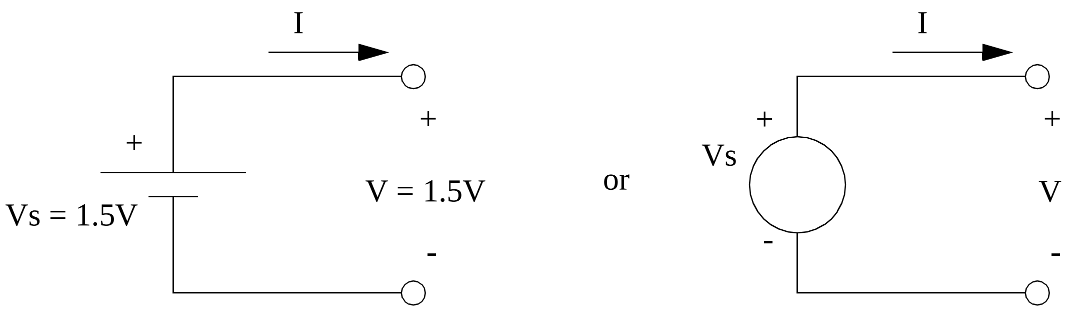
In some cases capacitors have polarity - this means that they must be connected so that the positive terminal is at a higher voltage. These devices will explode when connected backwards, and although small they can injure.

ASIDE: Capacitance is created by separating electrical charge by some distance. The actual component is made by having plates separated by a material called the dielectric (a non-conducting material). The area of the plates, and the distance between them are the main factors,

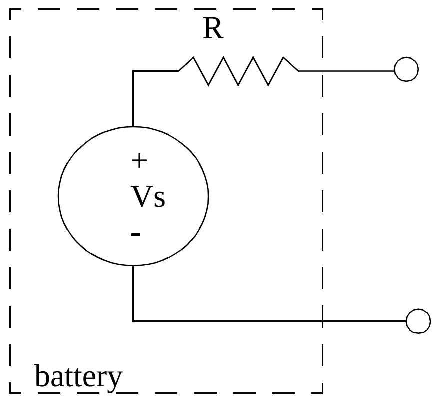


where,

• Voltage sources are also very common. Disposable batteries (e.g, 1.5V, 9V) are one good example. When we use these normally we assume that the batteries will supply any amount of current, at the rated voltage. The schematic symbol is shown below,

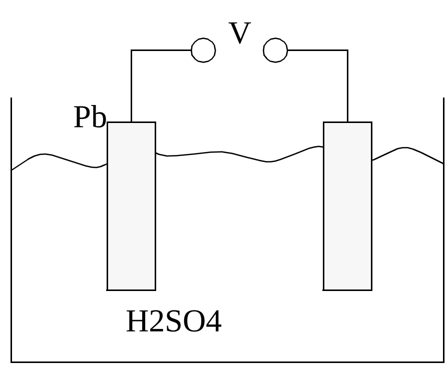


In reality batteries are often considered to have a small internal resistance. This reduces the voltages they provide when high currents are drawn,

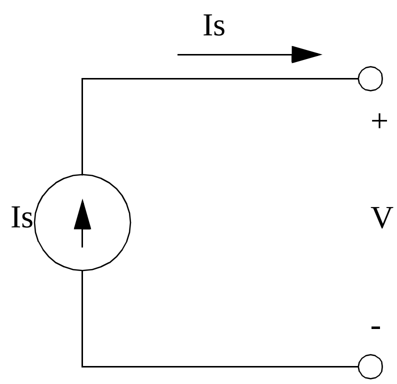


ASIDE: Most batteries are based on simple electrochemistry. The reaction equations for a lead acid battery show how electrons are generated by one reaction, and consumed by another. These two reactions occur separately at the positive and negative electrodes.

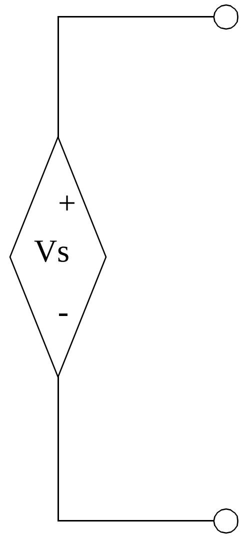
$$Pb + H_2SO_4 + HCl \rightarrow PbSO_4 + 2e^- + H_2$$



• A similar method is used when considering current sources



• Some theoretical treatments of circuit elements make use of dependant (variable) voltage and current sources. The schematic symbols are often as shown below,



$$V_s = \mu V_x = \rho I_x$$

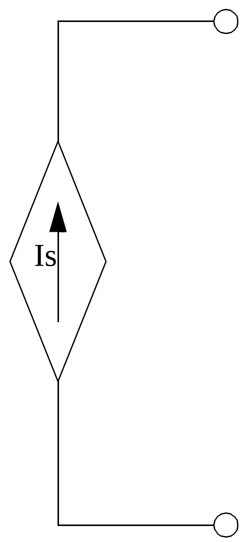
where,

V_s = the voltage of the dependant source

μ, ρ = coefficients

V_x = another voltage in the circuit

I_x = another current in the circuit



$$I_s = \alpha V_x = \beta I_x$$

where,

I_s = the current of the dependant source

α, β = coefficients

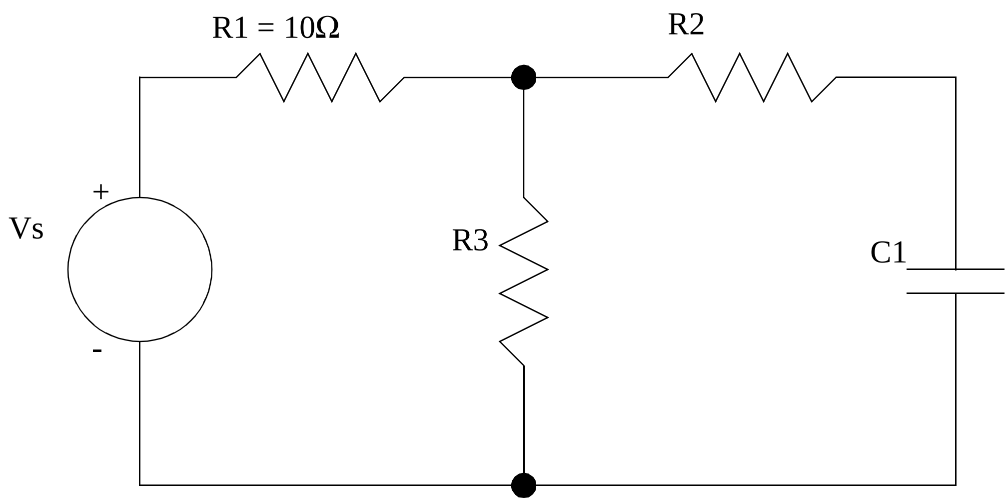
2.2 CIRCUIT DIAGRAMS

• Most of us will have seen a circuit diagram in the past, but there are some terms and conventions of importance when constructing and reading these diagrams.

• Generally,
  - we try to have positive voltages at the top, and sources on the left hand side.
  - each device has input/output terminals that are connected to nodes (black dots). They may not be drawn for simple connections, but are implied.
  - standard schematic symbols are available to reduce ambiguity.

- All components are labelled with variable names or values.

• Consider the example,



3. CIRCUIT ANALYSIS

• The techniques of circuit analysis focus on trying to derive equations that describe a circuit.

• In general most of the techniques attempt to simplify analysis by breaking the circuit into parts, loops, etc.

• Some well known techniques include,
  - mesh currents
  - node voltages
  - superposition
  - thevinen and norton equivalents

3.1 KIRCHOFF'S LAWS

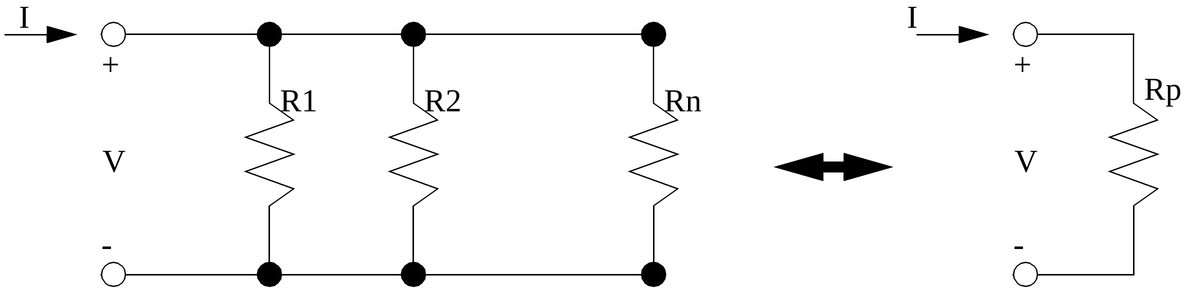
• Kirchoff's Current Law: "The sum of currents at any node in a circuit must equal zero" - keep in mind that current is a flow rate for moving electrons. And, electrons do not appear and disappear from the circuit. Therefore, all of the electrons flowing into a point in the circuit must be flowing back out.

• Kirchoff's Voltage Law: "The sum of all voltages about a closed loop in a circuit is equal to zero" - Each element will have a voltage (potential) between nodes. If any two points on the closed loop are chosen, and different paths chosen between them, the potentials must be equal or current will flow in a loop indefinately (Note: this would be perpetual motion).

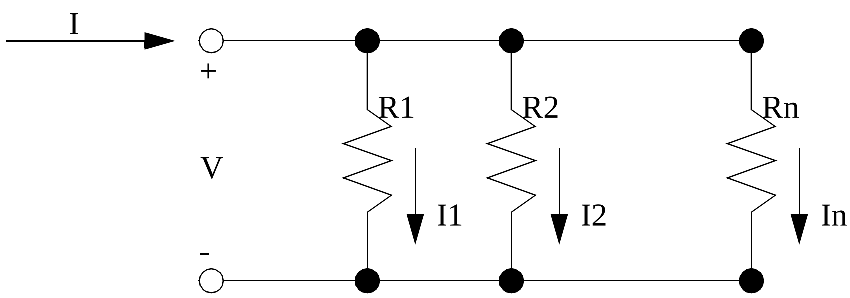
3.1.1 Simple Applications of Kirchoff's Laws

3.1.1.1 - Parallel Resistors

• Let's consider one on the most common electrical calculations - that for resistors in parallel. We want to find the equivalent resistance for the network of resistors shown.



First we can define currents in each branch of the circuit. Also recognize that the potential voltage across each resistor will be V.



Now, consider the sum of the currents in and out of the upper conductor,

$$\sum I = I - I_1 - I_2 - \ldots - I_n = 0$$

The current through each resistor is simple to calculate, so if we add the current through the resistors, and then relate the expression to Rp, the equation becomes,

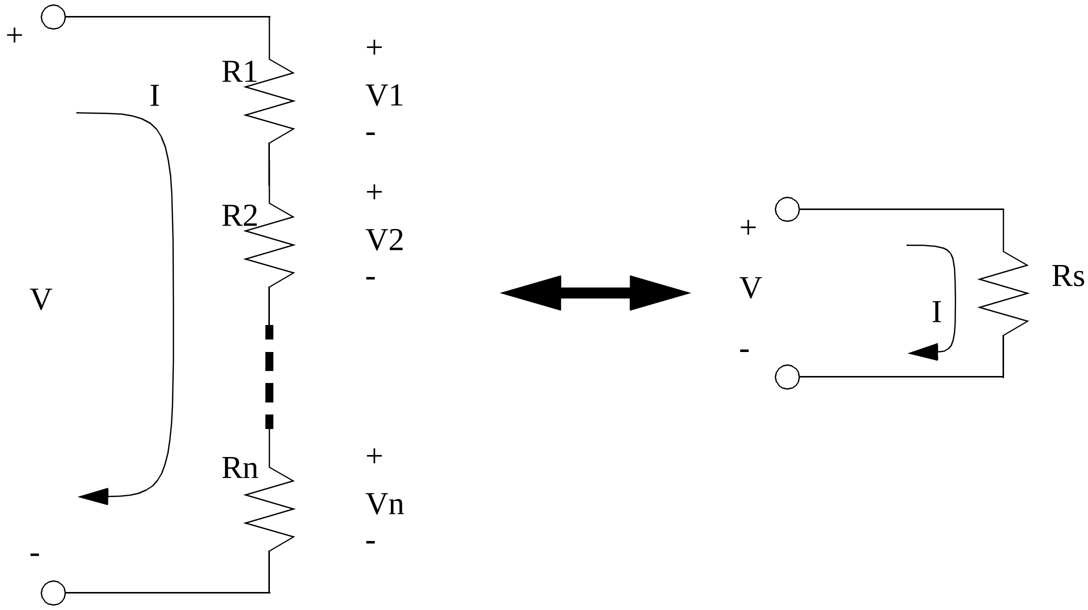
$$\therefore I - \frac{V}{R_1} - \frac{V}{R_2} - \ldots - \frac{V}{R_n} = 0$$

$$\therefore \frac{I}{V} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_n}\right) = \frac{1}{R_p}$$

$$\boxed{\therefore R_p = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_n}}}$$

3.1.1.2 - Series Resistors

- Now consider another problem with series resistors. We can use Kirchoff's voltage law to sum the voltages in the circuit loop. In this case the input voltage is a voltage rise, and the resistors are voltage drops (the signs will be opposite).



First, sum the voltages about the loop,

$$\sum V = -V + V_1 + V_2 + \ldots + V_n = 0$$

Next, use ohms law to replace voltages with the current, and then relate the values to the equivalent resistor Rs.

$$\therefore -V + IR_1 + IR_2 + \ldots + IR_n = 0$$

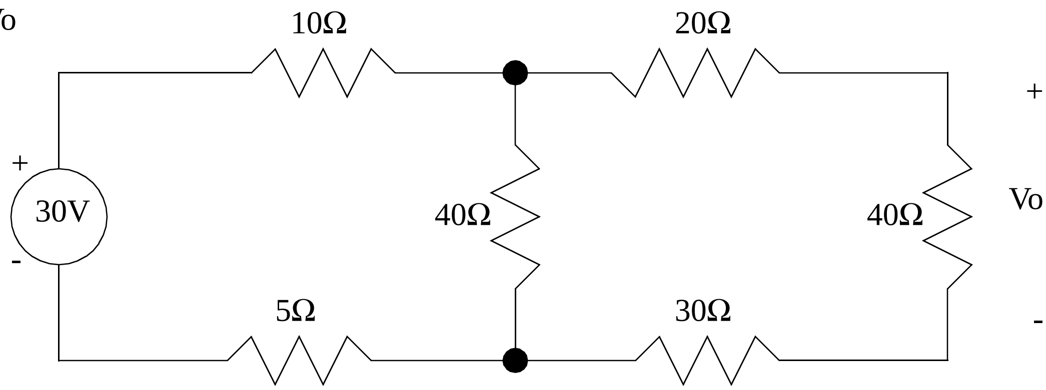
$$\therefore \frac{V}{I} = \boxed{R_1 + R_2 + \ldots + R_n = R_s}$$

ASIDE: We can use a “single subscript notation” to indicate a voltage or current. To do this we need to add a direction arrow for current, or use ‘+’ and ‘-’ for voltages.

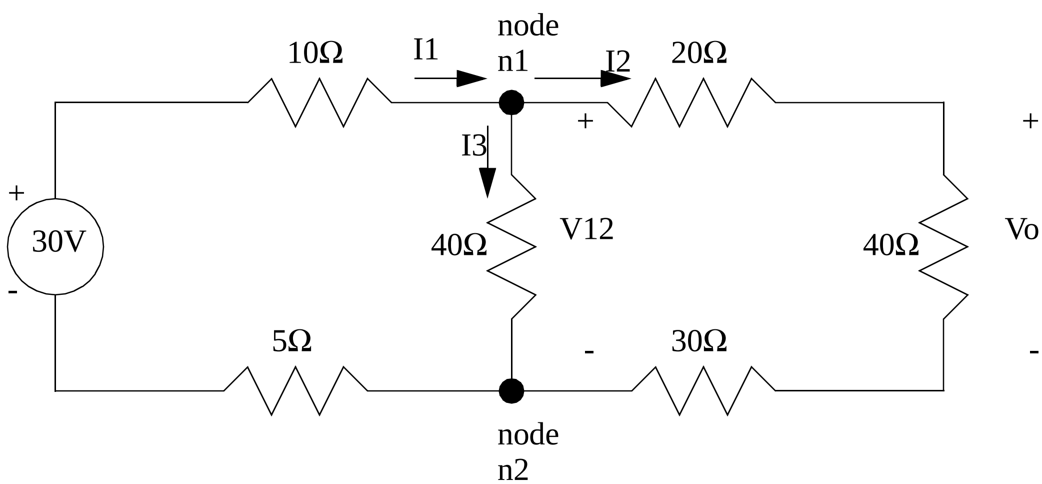
3.1.2 Node Voltage Methods

- If we consider that each conductor in a circuit has a voltage level, and that the components act as bridges between these, then we can try some calculations.
- This method basically involves setting variables, and then doing a lot of algebra.
- This is a very direct implementation of Kirchoff's current law.
- First, let's consider an application of The Node Voltage method for the circuit given below,

Find Vo



First, we will add labels for nodes, currents and voltages,



Next, sum the currents at node 1,

$$\sum I_{n_1} = I_1 - I_2 - I_3 = 0$$

Then find equations for the three currents based on the the difference between voltage at nodes 1 and 2. For the center path,

$$V_{12} = I_3 40\Omega \qquad\qquad \therefore I_3 = \frac{V_{12}}{40\Omega}$$

For the left branch,

$$V_{12} = -5I_1 + 30 - 10I_1 \qquad \therefore I_1 = \frac{V_{12} - 30}{-15} = \frac{30 - V_{12}}{15}$$

For the right branch,

$$V_{12} = 20I_2 + 40I_2 + 30I_2 \qquad \therefore I_2 = \frac{V_{12}}{90}$$

Now, combine the equations,

$$0 = I_1 - I_2 - I_3 = \frac{30 - V_{12}}{15} - \frac{V_{12}}{90} - \frac{V_{12}}{40\Omega}$$

$$\therefore V_{12} = 19.5V$$

Finally, find the current in the right circuit branch, and then the voltage across R2,

$$I_2 = \frac{V_{12}}{90} = \frac{19.5}{90} = 0.217A$$

$$V_{R_2} = I_2 40 = (0.217)(40) = 8.7V$$

ASIDE: When using a voltage between nodes we are using the double subscript notation. When doing this we label nodes and then the voltage is listed as ‘Vab’ where ‘a’ is positive relative to ‘b’.

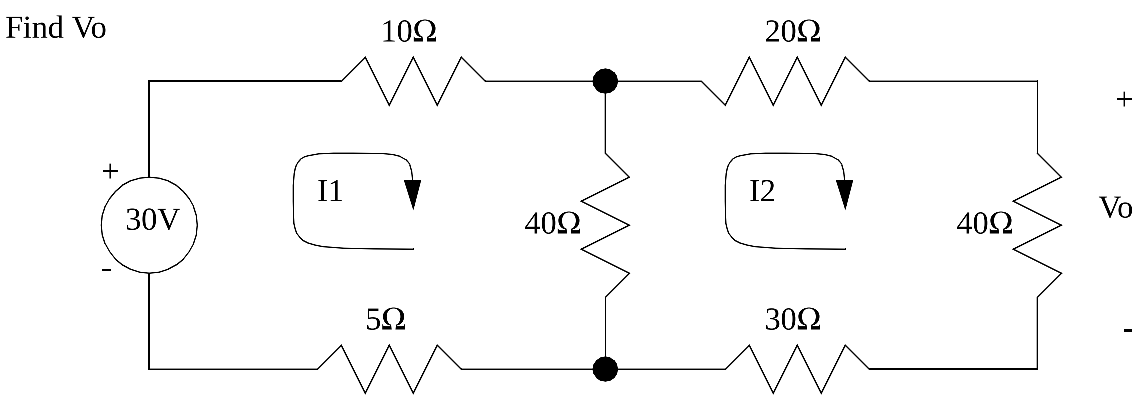
3.1.3 Current Mesh Methods

- If we consider Kirchoff’s Voltage law, we could look at any circuit as a collection of current loops. In some cases these current loops pass through the same components.

- We can define a loop (mesh) current for each clear loop in a circuit diagram. Each of these can be given a variable name, and equations can be written for each loop current.

• These methods are quite well suited to matrix solutions

• Lets consider a simple problem,



After adding the mesh currents (and directions) we may write equations,

$$\sum V_{I_1} = 0 = -30 + 10I_1 + 40(I_1 - I_2) + 5I_1$$

$$30 = 55I_1 - 40I_2 \qquad (1)$$

$$\sum V_{I_2} = 0 = 20I_2 + 40I_2 + 30I_2 + 40(I_2 - I_1)$$

$$0 = 40I_1 - 130I_2 \qquad (2)$$

Solve the equation matrix for I2, (I will use Cramer's Rule)

$$\begin{bmatrix} 55 & -40 & 30 \\ 40 & -130 & 0 \end{bmatrix} \qquad I_2 = \frac{det\begin{bmatrix} 55 & 30 \\ 40 & 0 \end{bmatrix}}{det\begin{bmatrix} 55 & -40 \\ 40 & -130 \end{bmatrix}} = \frac{55(0) - 30(40)}{55(-130) - (-40)(40)} = 0.216A$$

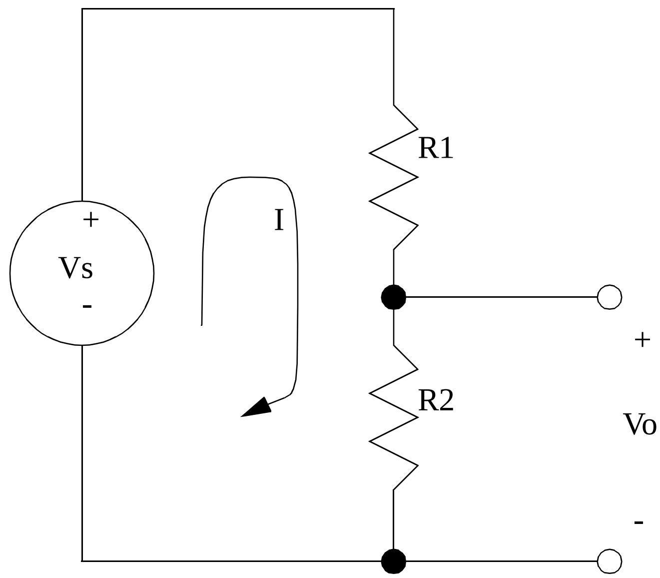
Finally calculate the voltage across the 40 ohm resistor,

$$V_O = 40I_2 = 8.6V$$

3.1.4 More Advanced Applications

3.1.4.1 - Voltage Dividers

• The voltage divider is a very common and useful circuit configuration. Consider the circuit below, we add a current loop, and assume there is no current out at Vo,



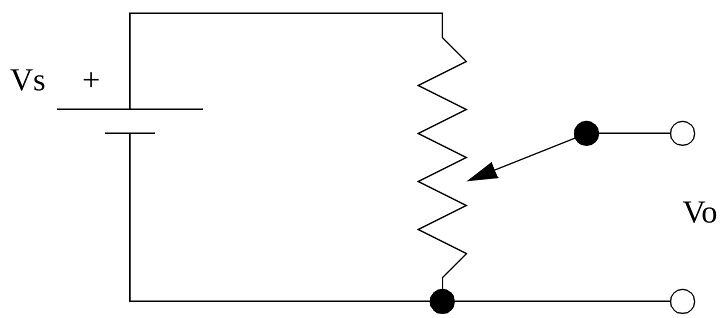
First, sum the voltages about the loop,

$$\sum V = -V_s + IR_1 + IR_2 = 0 \qquad \therefore I = \frac{V_s}{R_1 + R_2}$$

Next, find the output voltage, based on the current,

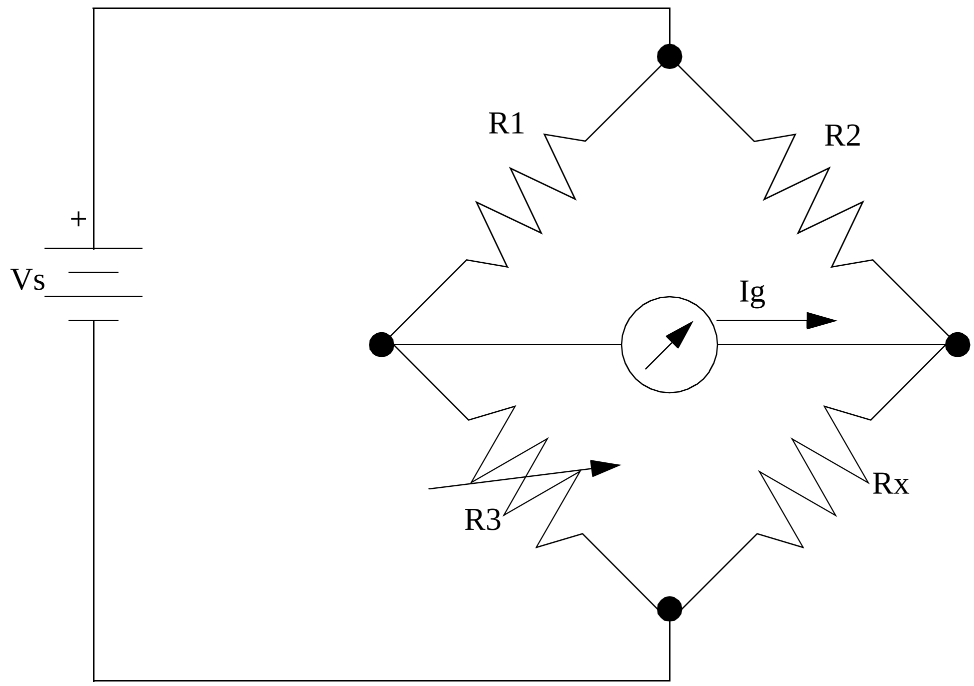
$$V_o = IR_2 = \left(\frac{V_s}{R_1 + R_2}\right)R_2 = \boxed{V_s\left(\frac{R_2}{R_1 + R_2}\right)}$$

ASIDE: variable resistors are often used as voltage dividers. As the wiper travels along the resistor the output voltage changes.



3.1.4.2 - The Wheatstone Bridge

• The wheatstone bridge is a very common engineering tool for magnifying and measuring signals. In this circuit a supply voltage Vs is used to power the circuit. Resistors R1 and R2 are generally equal, Rx is a resistance to be measured, and R3 is a tuning resistor. An ammeter is shown in the center, and resistor R3 is varied until the current in the center Ig is zero.



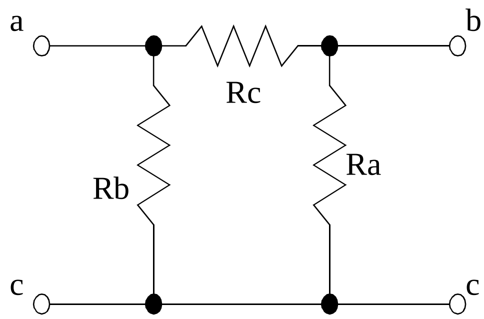
For practice try to show the relationship below holds for a balanced bridge (ie, Ig=0),

$$R_x = \frac{R_2 R_3}{R_1}$$

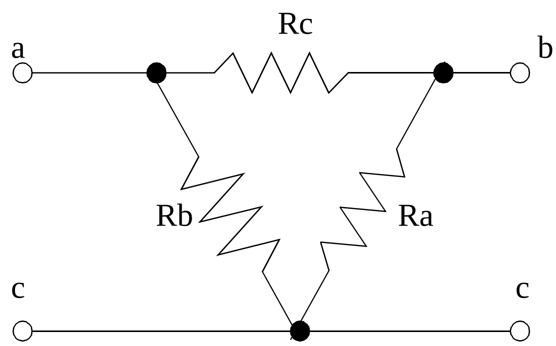
3.1.4.3 - Tee-To-Pi (Y to Delta) Conversion

- It is fairly common to use a model of a circuit. This model can then be transformed or modified as required.

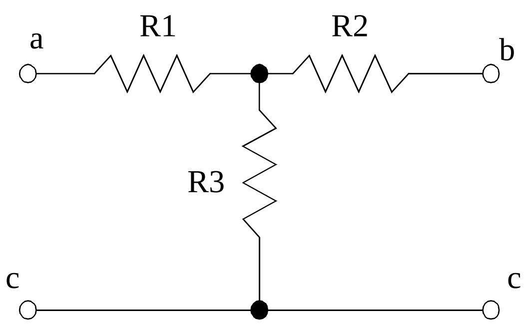
- A very common model and conversion is the Tee to Pi conversion in electronics. A similar conversion is done for power circuits called delta to y.



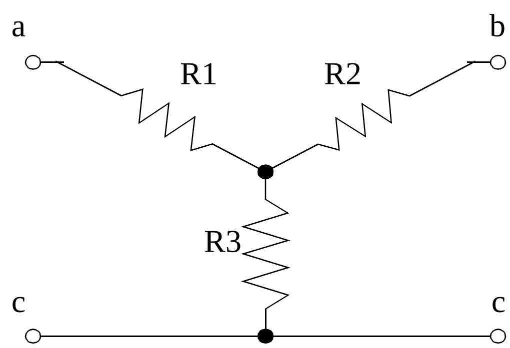
Pi



Delta



Tee



Y

- We can find equivalent resistors considering that,

$$R_{ac} = \frac{1}{\frac{1}{R_b} + \frac{1}{R_c + R_a}} = R_1 + R_3 \qquad \therefore \frac{R_b(R_c + R_a)}{R_a + R_b + R_c} = R_1 + R_3 \tag{1}$$

$$R_{ab} = \frac{1}{\frac{1}{R_c} + \frac{1}{R_a + R_b}} = R_1 + R_2 \qquad \therefore \frac{R_c(R_a + R_b)}{R_a + R_b + R_c} = R_2 + R_3 \tag{2}$$

$$R_{bc} = \frac{1}{\frac{1}{R_a} + \frac{1}{R_b + R_c}} = R_2 + R_3 \qquad \therefore \frac{R_a(R_b + R_c)}{R_a + R_b + R_c} = R_2 + R_3 \tag{3}$$

To find R1, (1)-(3)+(2)

$$\frac{R_b(R_c + R_a) - R_a(R_b + R_c) + R_c(R_a + R_b)}{R_a + R_b + R_c} = R_1 + R_3 - R_2 - R_3 + R_2 + R_3$$

$$\therefore \frac{R_bR_c + R_bR_a - R_aR_b - R_aR_c + R_cR_a + R_cR_b}{R_a + R_b + R_c} = 2R_1$$

$$\boxed{\therefore R_1 = \frac{R_bR_c}{R_a + R_b + R_c}}$$

Likewise,

$$\boxed{\begin{aligned} R_2 &= \frac{R_aR_c}{R_a + R_b + R_c} \\ R_3 &= \frac{R_aR_b}{R_a + R_b + R_c} \end{aligned}}$$

• To find the equivalents the other way,

$$R_a + R_b + R_c = \frac{R_b R_c}{R_1} = \frac{R_a R_c}{R_2} = \frac{R_a R_b}{R_3}$$

$$\therefore R_b = \frac{R_1}{R_2} R_a = \frac{R_3}{R_2} R_c$$

$$\therefore R_a = \frac{R_2}{R_1} R_b = \frac{R_3}{R_1} R_c$$

$$\therefore R_c = \frac{R_2}{R_3} R_b = \frac{R_1}{R_3} R_a$$

We can put these relationships into equation (1), to eliminate Ra and Rc,

$$\frac{R_b\left(\frac{R_2}{R_3}R_b + \frac{R_2}{R_1}R_b\right)}{\frac{R_2}{R_1}R_b + R_b + \frac{R_2}{R_3}R_b} = R_1 + R_3$$

$$\therefore \frac{R_b\left(\frac{R_2 R_1 + R_2 R_3}{R_1 R_3}\right)}{\frac{R_2 R_3 + R_1 R_3 + R_1 R_2}{R_1 R_3}} = R_1 + R_3$$

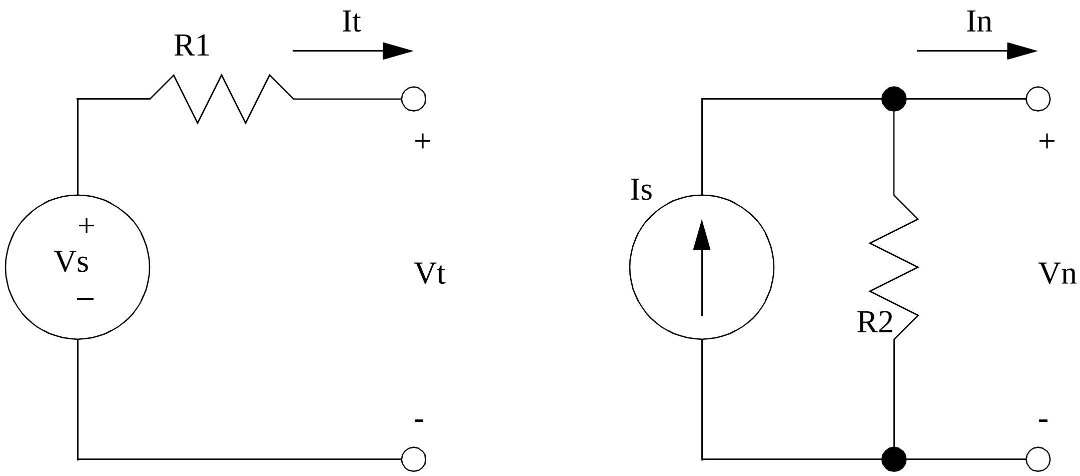
$$\therefore R_b = (R_1 + R_3)\left(\frac{R_2 R_3 + R_1 R_3 + R_1 R_2}{R_2 R_1 + R_2 R_3}\right) = \boxed{\frac{R_2 R_3 + R_1 R_3 + R_1 R_2}{R_2}}$$

Likewise,

$$\boxed{\begin{aligned} R_a &= \frac{R_2 R_3 + R_1 R_3 + R_1 R_2}{R_1} \\ R_c &= \frac{R_2 R_3 + R_1 R_3 + R_1 R_2}{R_3} \end{aligned}}$$

3.2 THEVENIN AND NORTON EQUIVALENTS

• A sometimes useful transformation is based on the equivalence of certain circuit elements,



Thevinen Norton

First, consider the short circuit case where,

$$I_t = I_n \qquad V_t = V_n = 0$$

this gives the basic relationship,

$$V_s = R_1 I_t \qquad \therefore \frac{V_s}{R_1} = I_t = I_n$$

Next, consider the open circuit case,

$$V_t = V_n \qquad I_t = I_n = 0$$

we find the simple relationships,

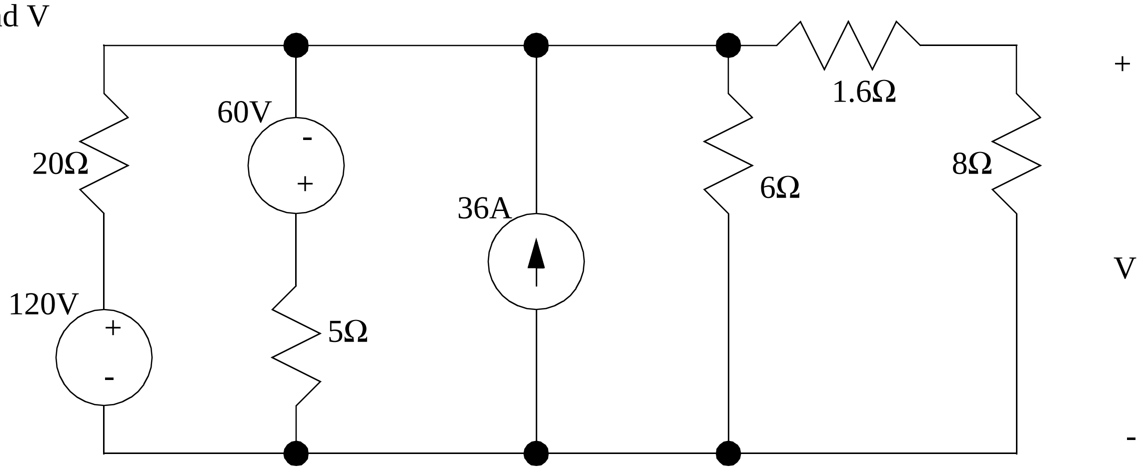
$$V_t = V_s \qquad V_n = I_s R_2$$

• We can use this to test an unknown circuit for open circuit voltage, and short circuit current, and then replace it with an equivalent circuit.

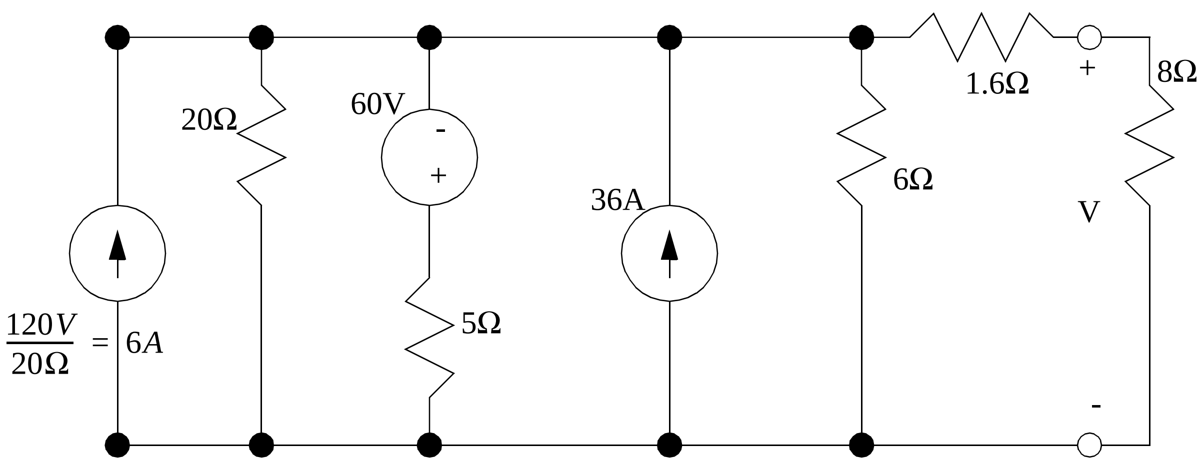
1. Measure open circuit voltage Vs
2. Measure short circuit current Is
3.a) If using a Thevenin equivalent calculate, Rs = Vs/Is
3.b) If using a Norton equivalent calculate, Rs = Vs/Is
4. Draw the appropriate circuit.
* note the resistor values are the same for both circuits.

• We can also use the Thevenin/Norton transformation to simplify circuits. Consider example 4.13 from [Nilsson].

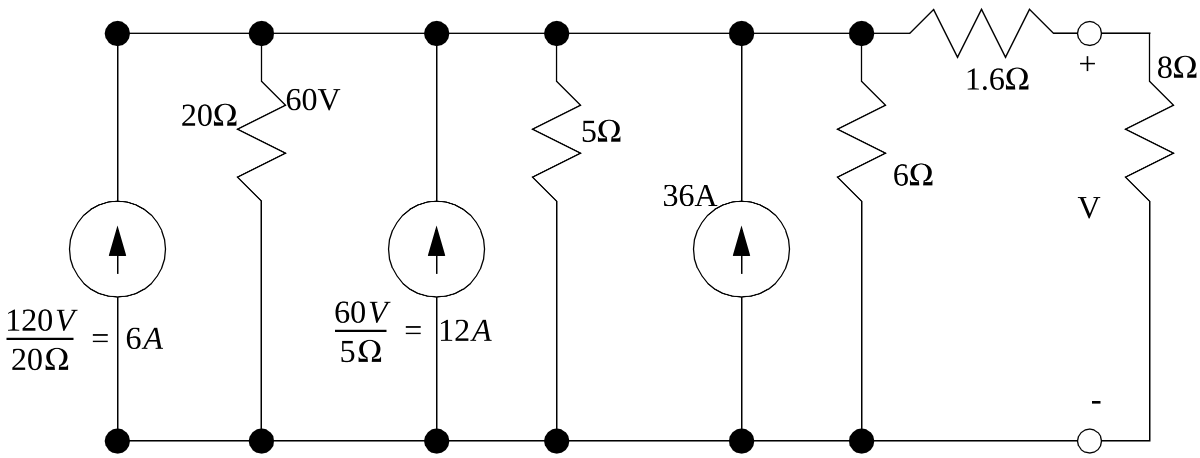
Find V



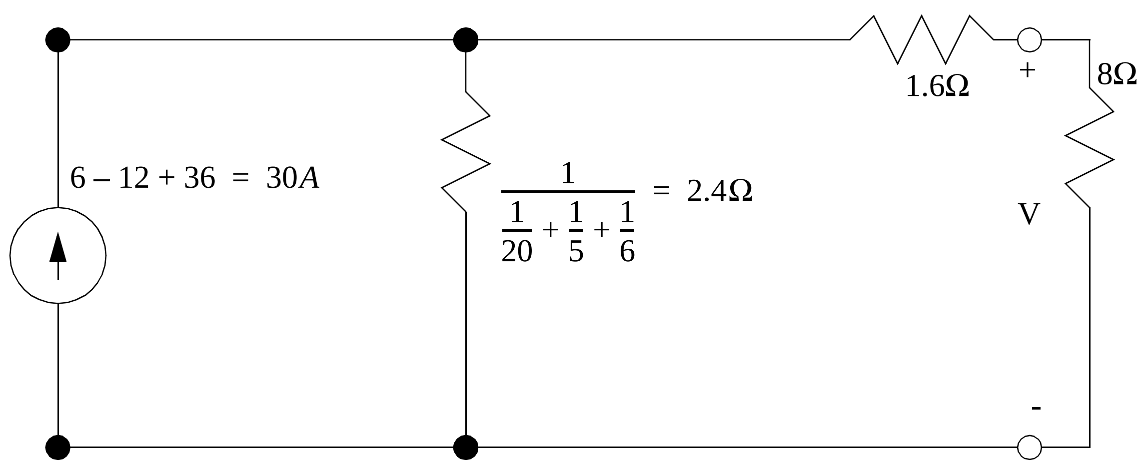
We can replace the first loop (120V source) with a Norton equivalent,



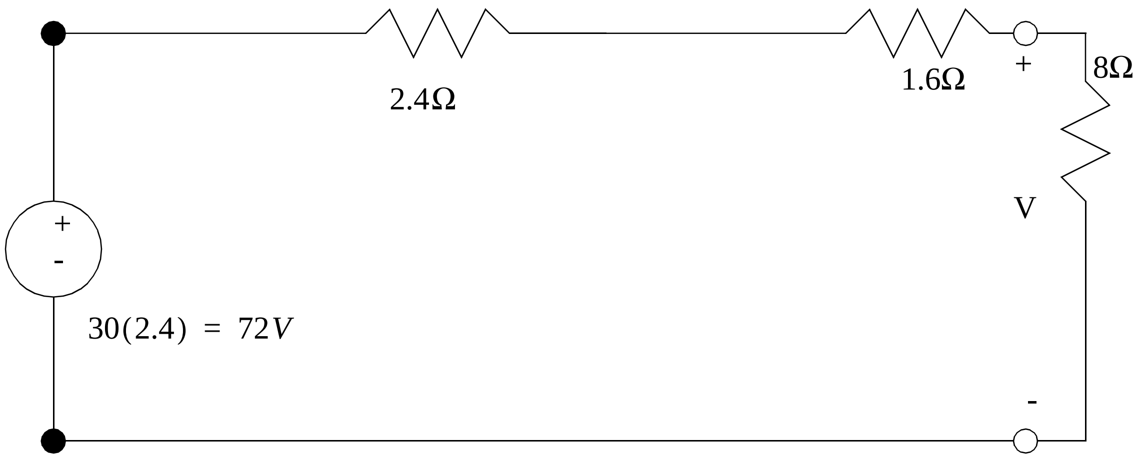
Next we can convert the remaining voltage source to a current source,



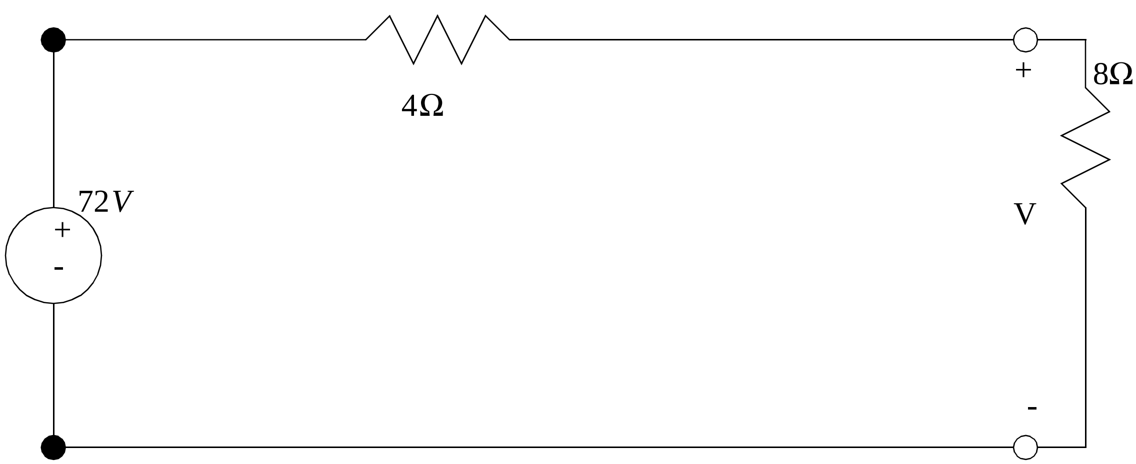
The current sources and resistors may now be combined to simplify the circuit,



Convert the current source to a voltage source using the Thevinen equivalent,



Combine the two serial resistors, and then use voltage division to find the output voltage.



$$V = 72\left(\frac{8}{8+4}\right) = 48V$$

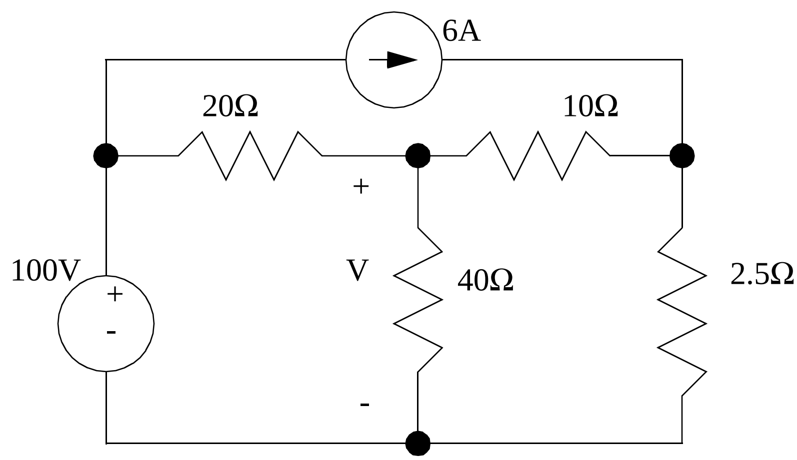
3.2.1 Superposition

• This is a simple technique that can be used when there are multiple sources in a circuit. The basic technique is,

1. Select one source in a circuit.
2. Make all other current sources open circuit.

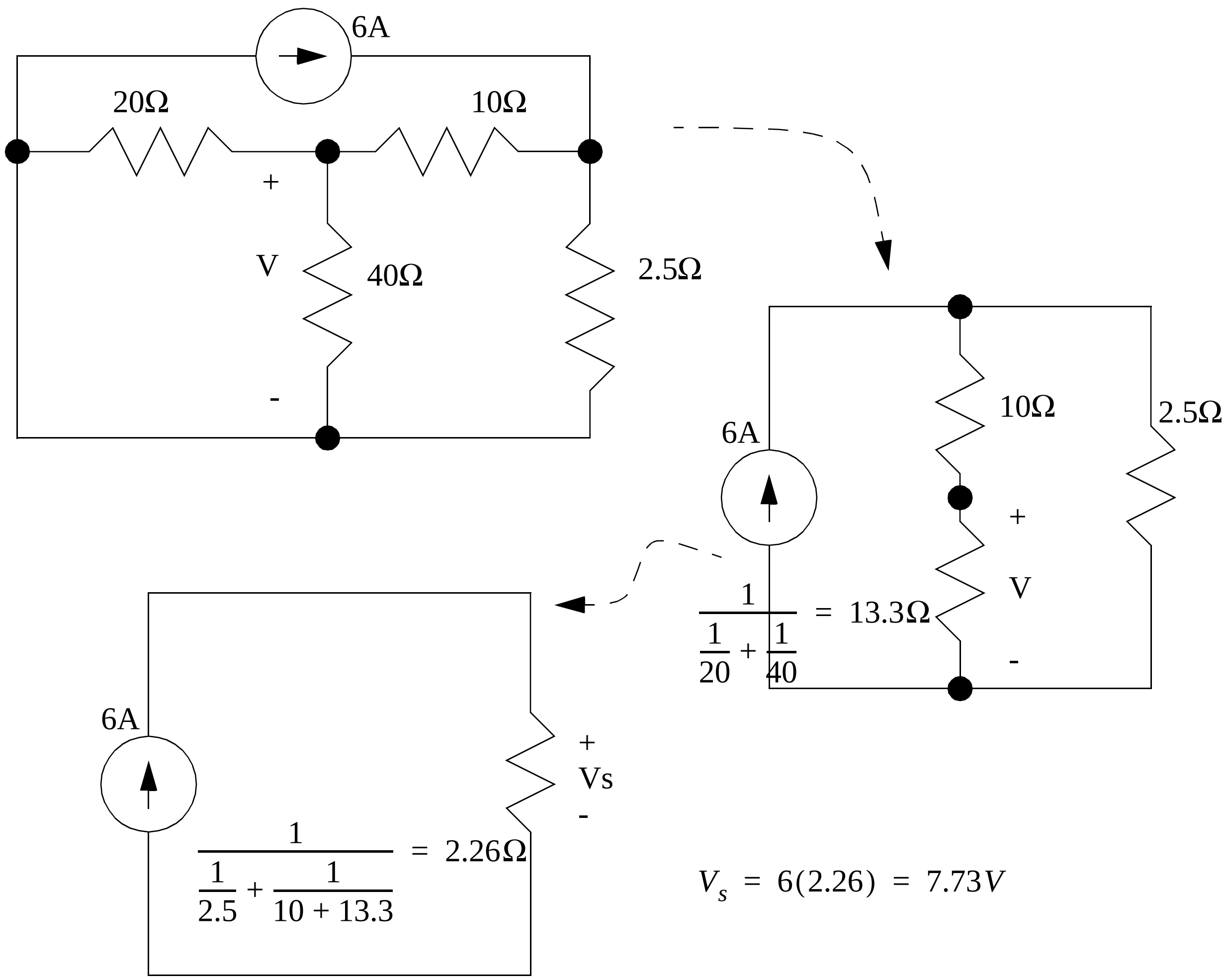
3. Make all other voltage sources short circuits.
4. Analyze as normal.
5. Pick the next voltage/current source and go back to 2.
6. Add together the results for each source.

• Consider an example below, 4.19 from [Nilsson],



Find the voltage V

a) Let's consider the current source first,

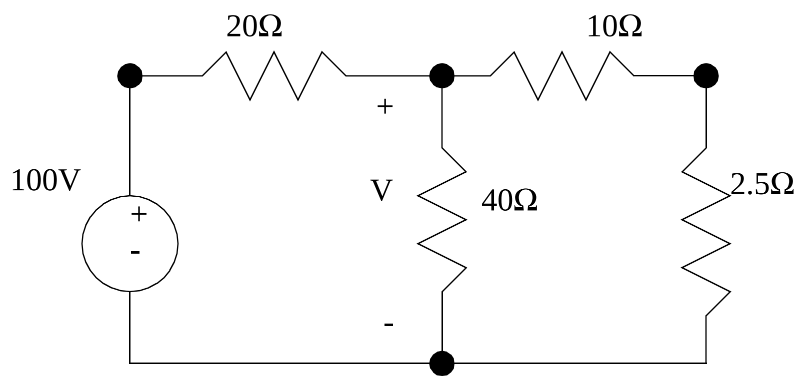


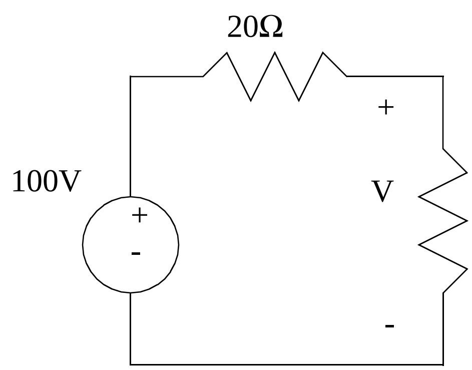
$$V_s = 6(2.26) = 7.73V$$

Find V using a voltage divider,

$$V = V_s\left(\frac{13.3}{10+13.3}\right) = 13.5V$$

b) Now find the effects of the voltage source with the current source as an open circuit,





$$\frac{1}{\frac{1}{40} + \frac{1}{10 + 2.5}} = 9.52\Omega$$

$$\therefore V = 100\left(\frac{9.52}{20 + 9.52}\right) = 32.25V$$

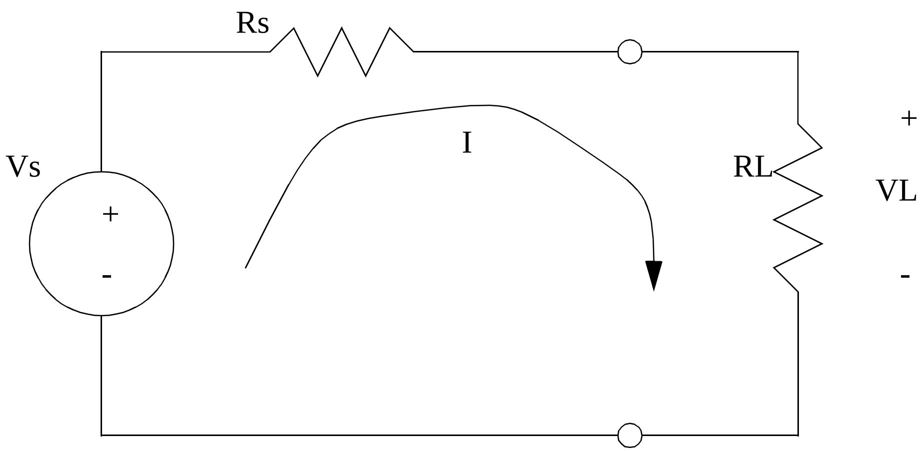
c) Finally we combine the effects of both sources,

$$\boxed{V = 7.73 + 32.25 = 40V}$$

3.2.2 Maximum Power Transfer

• When we will add a load to a network, we may want to try and maximize the amount of power delivered to it.

• Consider the simple case below,



Given Vs and Rs, find RL for the maximum power transfer.

$$I = \frac{V_s}{R_s + R_L} \qquad V = IR_L$$

$$P_L = IV_L = \left(\frac{V_S}{R_S + R_L}\right)^2 R_L$$

$$\frac{\partial}{\partial R_L} P_L = 0$$

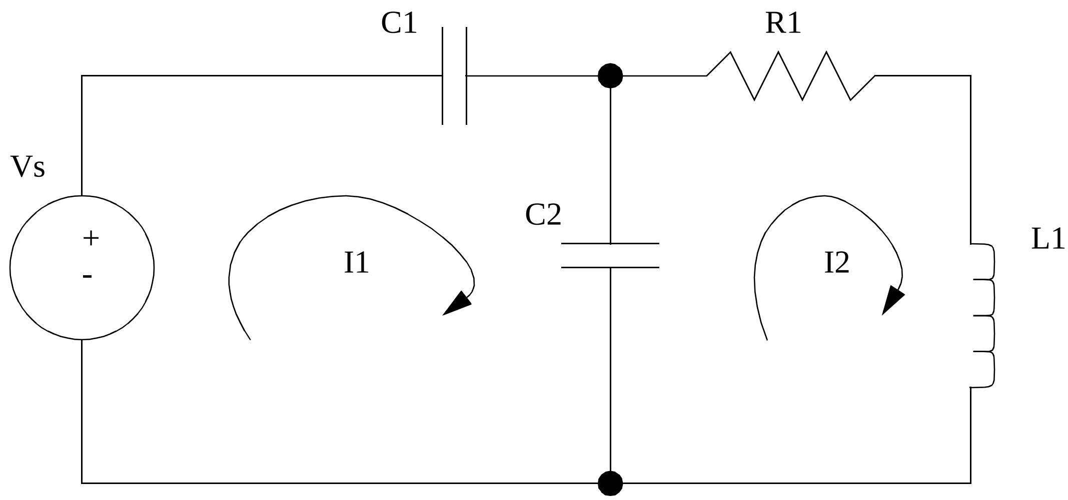
$$\boxed{\therefore R_L = R_S}$$ For maximum power transfer

• For practice try proving this theorem for the Norton equivalent circuit.

3.3 CIRCUITS CONTAINING CAPACITORS AND INDUCTORS

• When circuits contain capacitors, inductors, or other complex components, the solving techniques are the same. The main difference is that you will end up with differential expressions. (later we will see better techniques for dealing with these components)

• Consider the example,



For the two loops,

$$\sum V_{I_1} = -V_S + C_1\int I_1 dt + C_2\int (I_1 - I_2)dt = 0$$

$$\therefore V_S = (C_1 + C_2)\int I_1 dt - C_2\int I_2 dt$$

$$\therefore \int I_1 dt = \frac{V_S + C_2\int I_2 dt}{C_1 + C_2} \qquad (1)$$

$$\sum V_{I_2} = C_2\int (I_2 - I_1)dt + R_1 I_2 + L\frac{d}{dt}I_2 = 0 \qquad (2)$$

sub. (1) into (2),

$$-C_2\left(\frac{V_S + C_2\int I_2 dt}{C_1 + C_2}\right) + C_2\int I_2 dt + R_1 I_2 + L\frac{d}{dt}I_2 = 0$$

etc.......

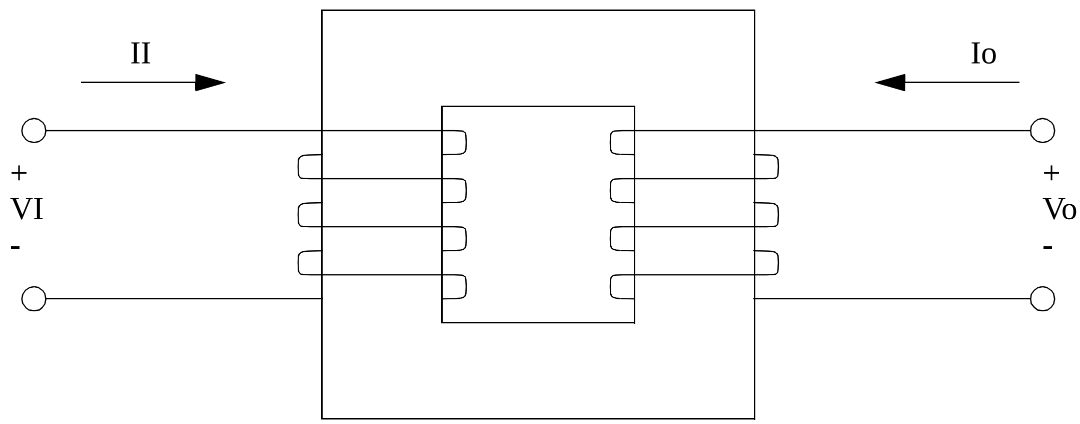
4. PASSIVE DEVICES

• Passive devices will have the same operating characteristics at the same operating points.

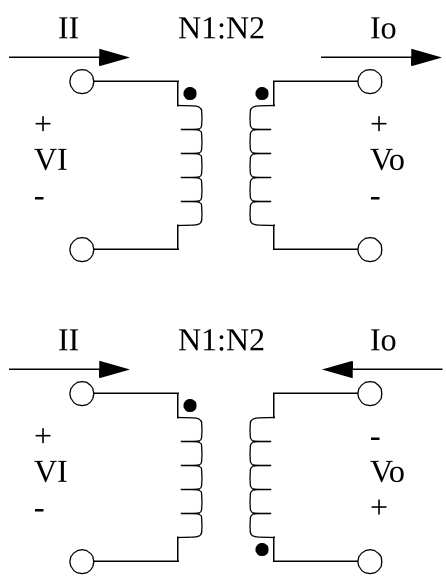
4.1 TRANSFORMERS

• A transformer can be viewed as a converter that can increase voltage and lower current, or vice versa. It only works when using AC.

• The transformer is effectively a magnetic circuit. The transformer has two or more coils of wire wrapped about a common core.



• The ideal relationship is,

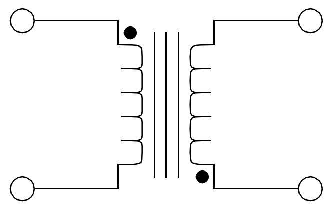


$$\frac{V_I}{N_I} = \frac{V_o}{N_o} \qquad \text{and} \qquad I_I N_I = I_o N_o$$

where,

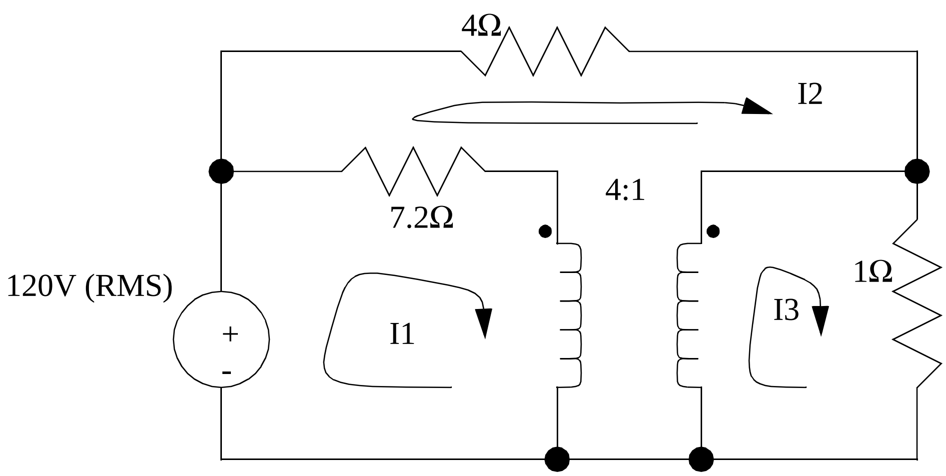
N1 = the number of coils on the primary side
N2 = the number of coils on the secondary side

• If a transformer has an iron core it will be shown with lines in the centre,



- To deal with a transformer in a circuit analysis we need to pay attention to the polarity of the coils, and we may consider the inductance of each coil at times.

- Consider the example below, from [Nilsson, pg. 450]. We want to find the power delivered to the 1ohm resistor. We will use the mesh current method,



For I1,

$$120V = 7.2(I_1 - I_2) + V_t \qquad (1)$$

For I2,

$$4I_2 + \frac{V_t}{4} - V_t + 7.2(I_2 - I_1) = 0 \qquad (2)$$

For I3,

$$1I_3 - \frac{V_t}{4} = 0 \qquad \therefore V_t = 4I_3 \qquad (3)$$

(3) into (1), $120 = 7.2I_1 - 7.2I_2 + 4I_3$

(3) into (2), $0 = -7.2I_1 + 11.2I_2 - 3I_3$

For the transformer,

$$4(I_1 - I_2) = 1(I_3 - I_2)$$

$$I_1 - 3I_2 - I_3 = 0 \qquad (4)$$

Next we can solve the remaining three equations and three unknown currents using a matrix approach,

$$\begin{bmatrix} 7.2 & -7.2 & 4 \\ -7.2 & 11.2 & -3 \\ 4 & -3 & -1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 120 \\ 0 \\ 0 \end{bmatrix}$$

$$I_3 = \frac{\begin{bmatrix} 7.2 & -7.2 & 120 \\ -7.2 & 11.2 & 0 \\ 4 & -3 & 0 \end{bmatrix}}{\begin{bmatrix} 7.2 & -7.2 & 4 \\ -7.2 & 11.2 & -3 \\ 4 & -3 & -1 \end{bmatrix}} = \frac{-2784}{-100} = 27.8$$

Finally we find the power in the resistor,

$$P = 1\Omega(44.1A) = \boxed{44W}$$

5. ACTIVE DEVICES

• Active devices are different from passive devices such as resistors, capacitors and inductors. These devices are capable of changing their operational performance, may deliver power to the circuit, and can perform interesting mathematical functions.

• When doing most circuits problems we depends on idealized components. The following section will describe a number of components, thier models, and how to apply them in practical circuits.

5.1 OPERATIONAL AMPLIFIERS

• A very common and versatile device is the operational amplifier (Op-amp). They are characterized as,

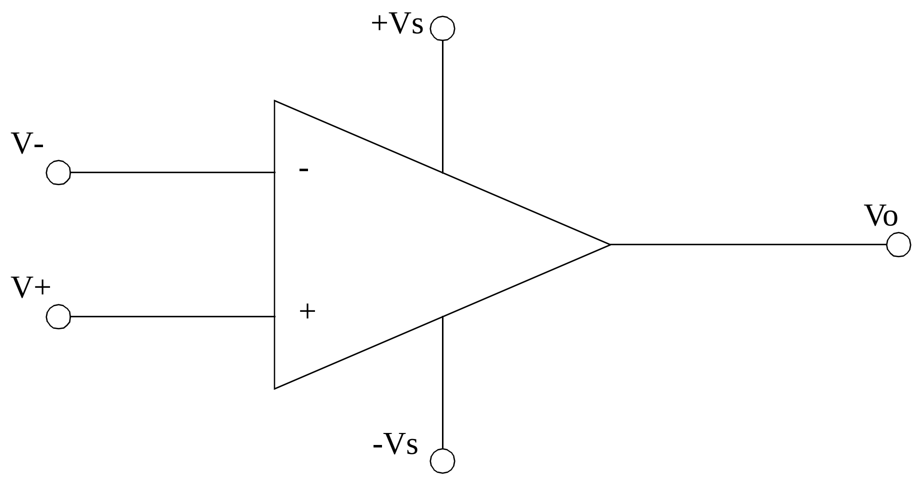
- stable high gain amplifiers
- high input impedances

- low output impdeances

• These are available for a few cents in commercial quantities. They also come in a wide variety of packages for various applications.

5.1.1 General Details

• The schematic symbol for these devices is given below,



• Inside these devices have a very high gain amplifier that compares the inputs and gives an output that is amplified as shown by,

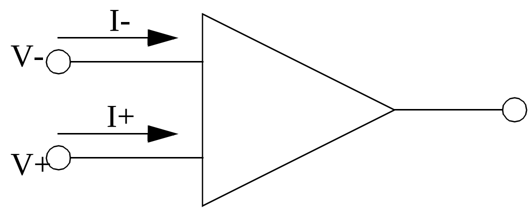
$$V_o = G(V_+ - V_-)$$

where,

V- is the inverting input
V+ is the non-inverting input
G is the gain (typically 100000x)
Vo is the output
Vs is the supply voltage

$|V_o| \le V_s$ — If the output voltage is pushed beyond +Vs or -Vs, then it will be clipped at, or slightly below the source voltage.

• When using these devices the circuit is typically set up so that both the inverting and non-inverting inputs have the same voltage, and the currents in to both of the inputs is negligable.



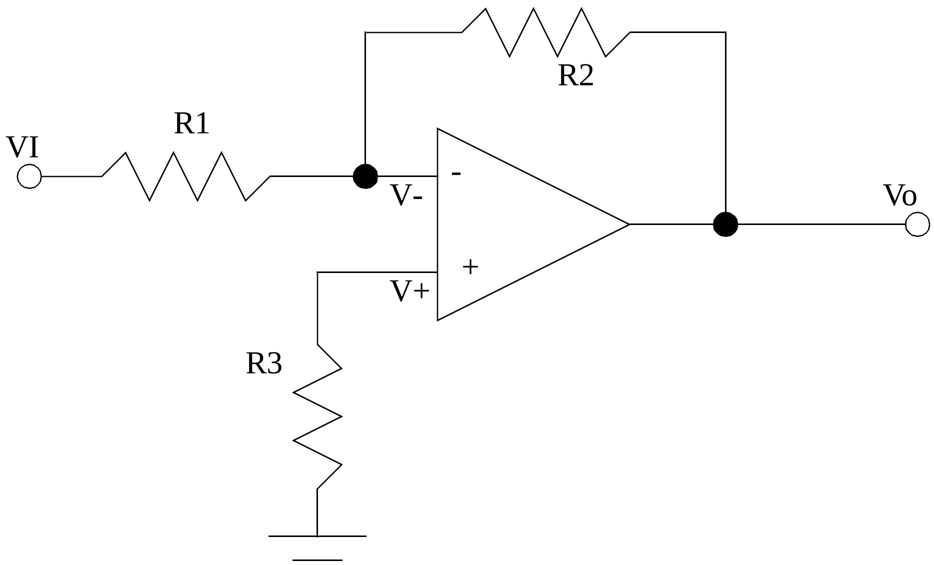
V+ = V-   I+ = I- = 0

5.1.2 Simple Applications

• Considering that the Op-amp was originally designed to allow simple mathematical operations in circuit form, the following circuits tend to bemathematical in nature.

5.1.2.1 - Inverting Amplifier

• A typical op-amp application is the inverting amplifier.



R1 and R2 are effectively a voltage divider,

$$V_- = (V_I - V_O)\left(\frac{R_2}{R_1 + R_2}\right) + V_o$$

The circuit tries to keep both op-amp inputs equal. And, the V+ input is grounded.

$$V_- = V_+ = 0$$

$$\therefore 0 = V_I\left(\frac{R_2}{R_1 + R_2}\right) + V_o\left(1 - \frac{R_2}{R_1 + R_2}\right)$$

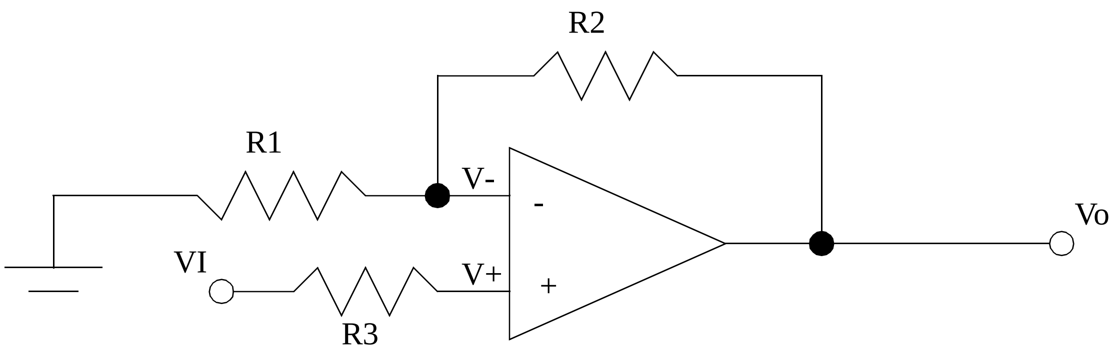
$$\therefore V_o\left(\frac{R_1 + R_2 - R_2}{R_1 + R_2}\right) = -V_I\left(\frac{R_2}{R_1 + R_2}\right)$$

$$\boxed{\therefore \frac{V_o}{V_I} = -\frac{R_2}{R_1}}$$

Amplifier Gain

<u>5.1.2.2 - Non-Inverting Amplifier</u>

• We can also make non-inverting amplifiers using the following circuit,



Use R1 and R2 as a voltage divider,

$$V_- = V_o\left(\frac{R_1}{R_1 + R_2}\right)$$

If we consider the the two inputs to have an equivalent input voltages,

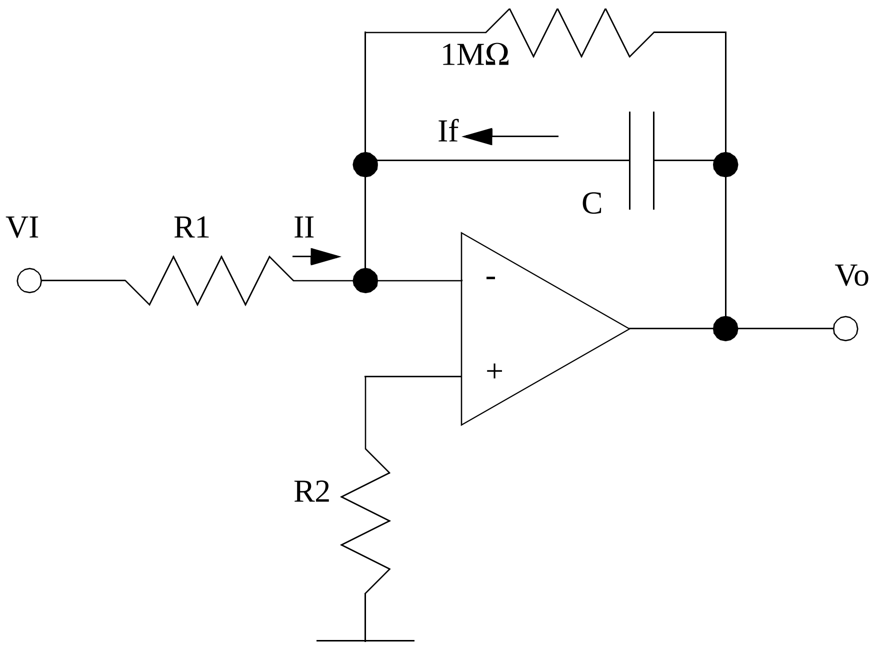
$$V_- = V_+ = V_I$$

$$\therefore V_I = V_O\left(\frac{R_1}{R_1 + R_2}\right)$$

$$\boxed{\therefore \frac{V_O}{V_I} = \frac{R_1 + R_2}{R_1}}$$ Gain

5.1.2.3 - Integrator

• The integrating amplifier is a very powerful application,



In the circuit the 1MΩ resistor is used to prevent output drift. Recall that in this configuration,

$$V_- = V_+ = 0$$

We can then find the currents,

$$I_I = \frac{V_I}{R_1} \qquad I_f = C\frac{d}{dt}V_o$$

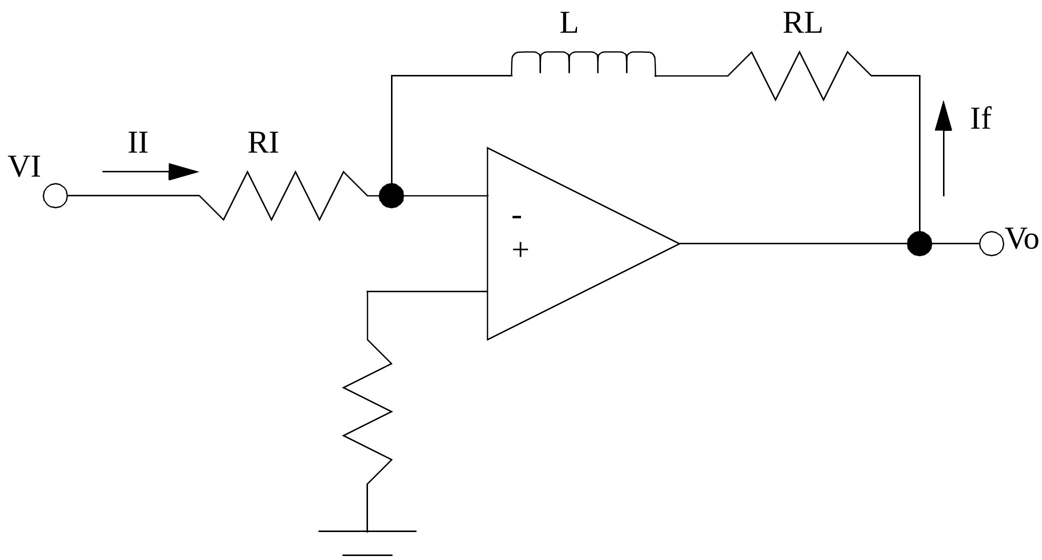
Finally,

$$\sum I_{V_-} = I_I - I_f = 0 = \frac{V_I}{R_1} - C\frac{d}{dt}V_o$$

$$\boxed{\therefore V_o = \frac{1}{R_1 C}\int V_I dt}$$

5.1.2.4 - Differentiator

• The following device is one form of differentiator using an inductor,



We may begin by realizing that the two op-amp inputs are at zero volts.

$$V_- = V_+ = 0$$

Next the input, and feedback currents may be found and summed,

$$I_I = \frac{V_I}{R_I} \qquad I_f = \frac{V_o}{L\frac{d}{dt} + R_L}$$

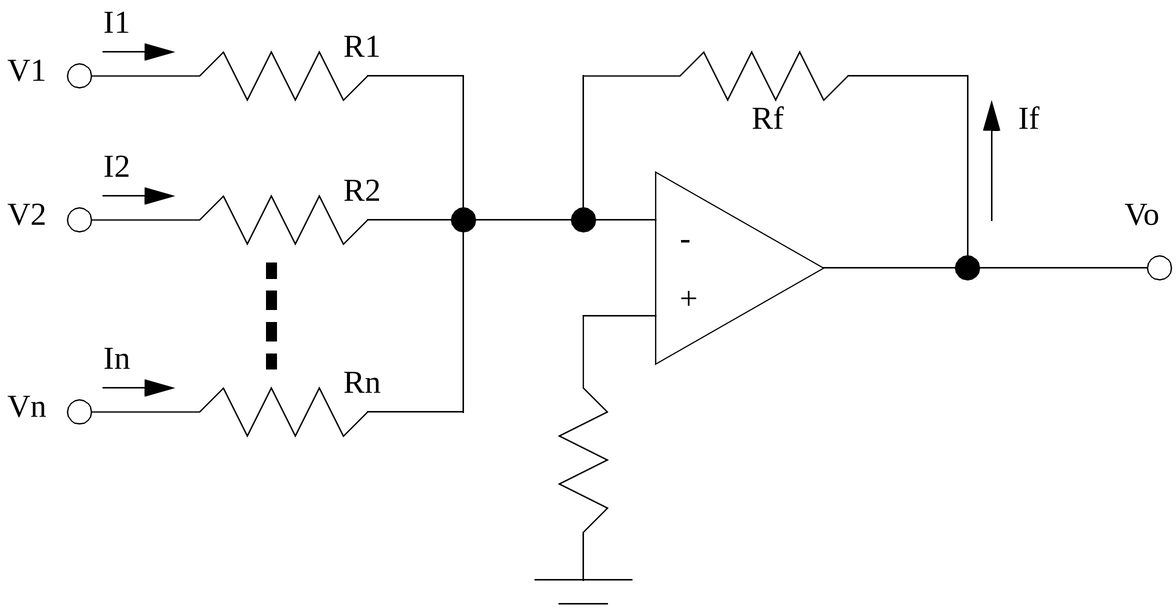
$$\sum I = I_I - I_f = 0 = \frac{V_I}{R_I} - \frac{V_o}{L\frac{d}{dt} + R_L}$$

$$\boxed{\therefore V_o = V_I\left(\frac{R_L}{R_I} + \frac{L\frac{d}{dt}}{R_I}\right)}$$

• A second type of circuit uses a capacitor to find the differential,

5.1.2.5 - Weighted Sums

• The following circuit can be used to add inputs. If dissimilar components are used the inputs can be weighted



We can recognize that the op-amp inputs are both kept at zero volts, and that there is not current into the non-inverting input, we can find,

$$V_- = V_+ = 0$$

$$I_1 = \frac{V_1}{R_1} \qquad I_2 = \frac{V_2}{R_2} \qquad I_n = \frac{V_n}{R_n}$$

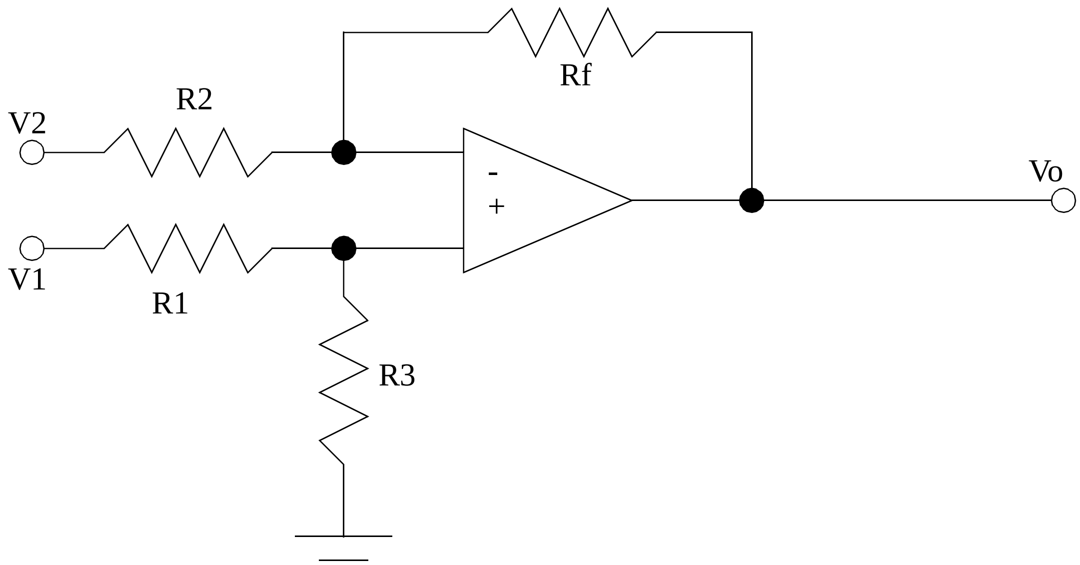
$$I_f = \frac{V_o}{R_f}$$

$$\sum I = I_1 + I_2 + I_n - I_f = 0 = \frac{V_1}{R_1} + \frac{V_2}{R_2} + \ldots + \frac{V_n}{R_n} - \frac{V_o}{R_f}$$

$$\boxed{\therefore V_o = R_f\left(\frac{V_1}{R_1} + \frac{V_2}{R_2} + \ldots + \frac{V_n}{R_n}\right)}$$

5.1.2.6 - Difference Amplifier (Subtraction)

• We can construct an amplifier that subtracts one input from the other,



Using the normal approach, we can see that both inputs are essentially voltage dividers,

$$V_+ = V_1\left(\frac{R_3}{R_1 + R_3}\right)$$

$$V_- = (V_2 - V_o)\left(\frac{R_f}{R_2 + R_f}\right) + V_o$$

Next, we can set the two inputs equal, and combine the equations,

$$V_+ = V_- = V_1\left(\frac{R_3}{R_1 + R_3}\right) = (V_2 - V_o)\left(\frac{R_f}{R_2 + R_f}\right) + V_o$$

$$\therefore V_o\left(\frac{R_2 + R_f - R_f}{R_2 + R_f}\right) = V_1\left(\frac{R_3}{R_1 + R_3}\right) - V_2\left(\frac{R_f}{R_2 + R_f}\right)$$

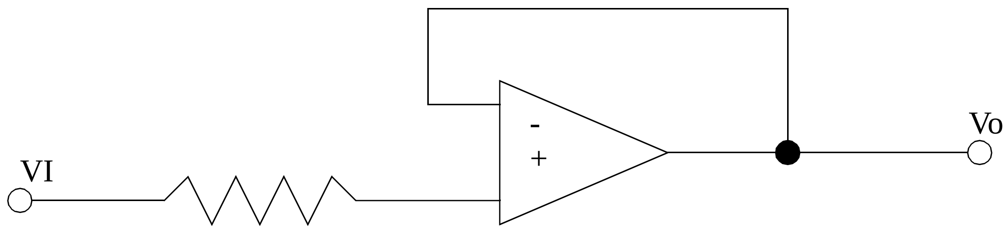
$$\boxed{\therefore V_o = V_1\left(\frac{R_3(R_2 + R_f)}{R_2(R_1 + R_3)}\right) - V_2\left(\frac{R_f}{R_2}\right)}$$

Note the result if all resistor values are equal,

$$V_o = V_1 - V_2$$

5.1.2.7 - Op-Amp Voltage Follower

• At times we want to isolate a voltage source from an application, or add a high impedance. This can be done using a voltage follower,



We can develop some of the basic relationship for this circuit,

$$V_+ = V_I$$

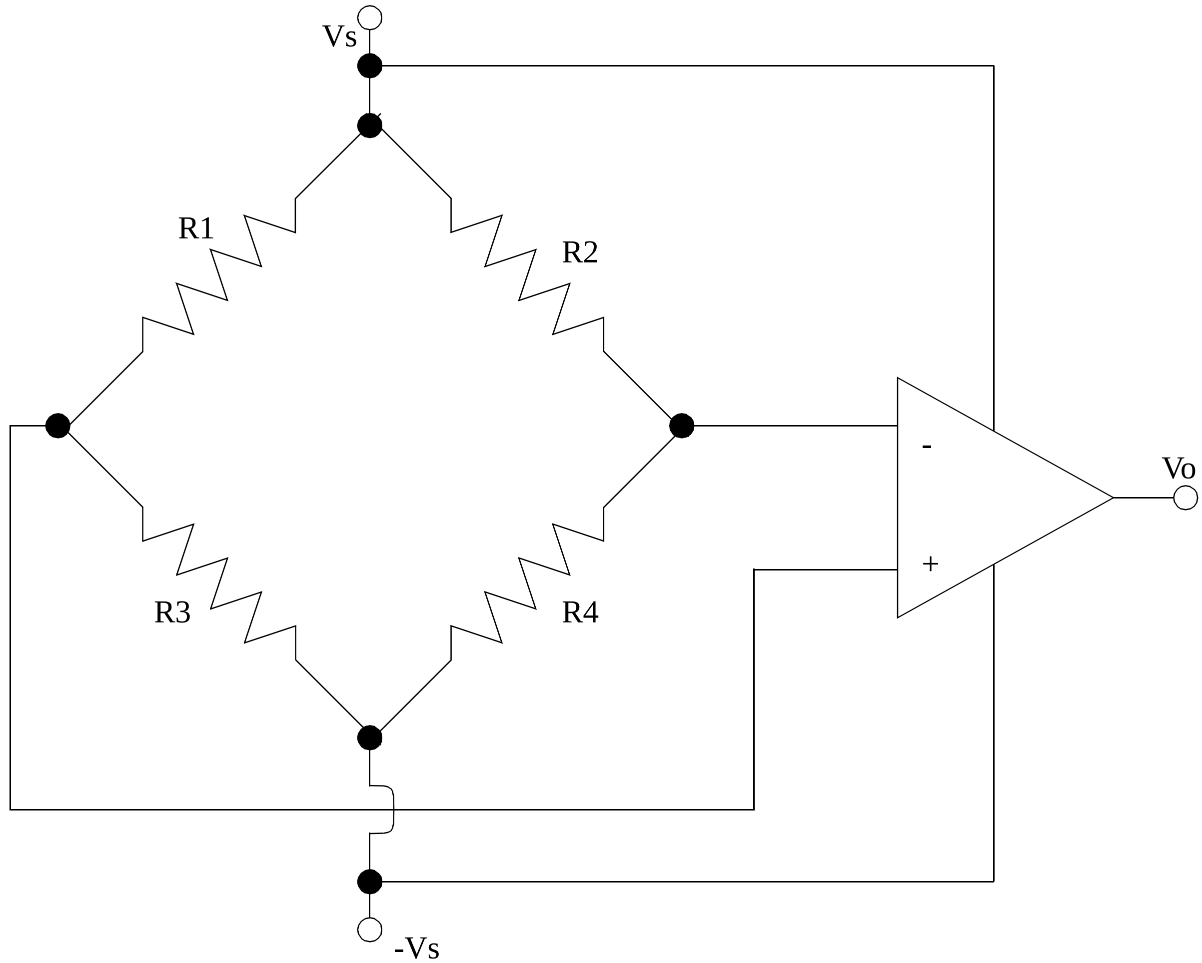
$$V_+ = V_-$$

$$V_o = V_-$$

$$\boxed{\therefore V_o = V_I}$$

5.1.2.8 - Bridge Balancer

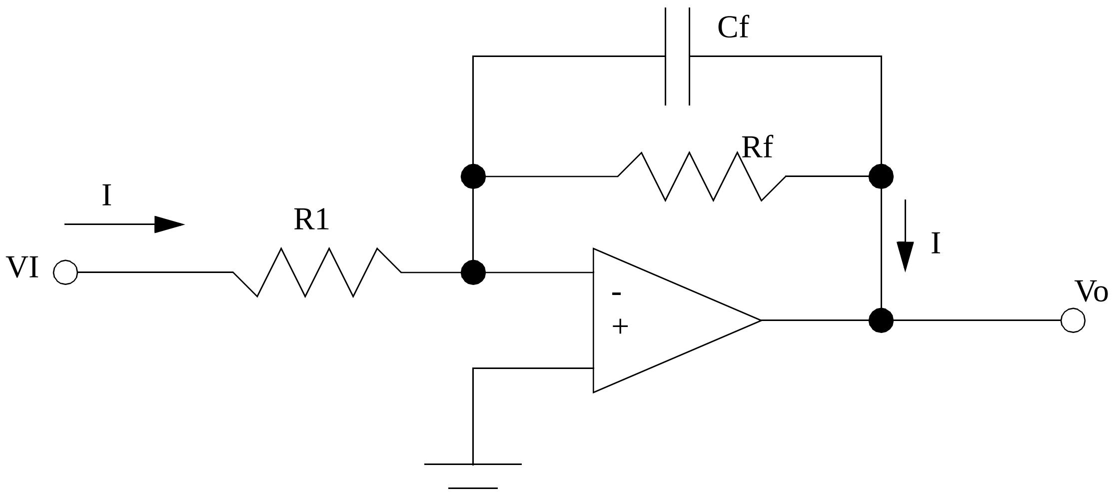
• Op-amps can be used for measuring the potential across bridges.



• When used in this mode, it is probable that both inputs may have the same voltage that is not zero. The result of this common offset is that the output will drift with the common inputs. The technical measure is the Common Mode Rejection Ratio (CMRR). This is generally measured by the manufacturer, and provided in the device specifications.

5.1.2.9 - Low Pass Filter

• A Low pass filter will enable us to cut off the higher frequency components of an input signal,



Recognizing that the inverting input is at ground we may write the following expressions for the current,

$$V_I = IR_1 \qquad V_o = -I\left(\frac{1}{\frac{1}{R_f} + j\omega C_f}\right)$$

Next, we can combine these expressions to find the gain of the system,

$$I = \frac{V_I}{R_1} = \frac{-V_o}{\left(\frac{R_f}{1 + j\omega C_f R_f}\right)}$$

$$\boxed{\therefore \frac{V_o}{V_I} = \frac{-R_f}{R_1 + R_1 j\omega C_f R_f}}$$

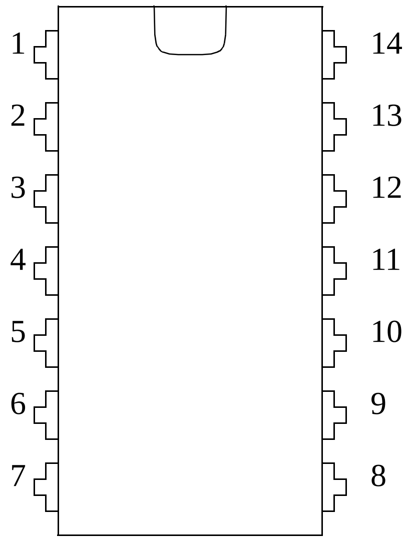
NOTE: The gain will drop as the frequency rises. Past a certain point the gain will be very low. The corner frequency and gain are,

$$gain = -\frac{R_f}{R_1}$$

$$\omega_c = \frac{1}{C_f R_f}$$

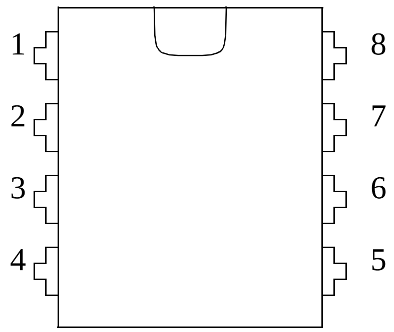
5.1.2.10 - The 741 Op-Amp

• The basic layout of the 741 op-amp is given below for the 14 pin dip package (14C1741).



Pin #	Description
1	no connection
2	no connection
3	offset control
4	V- (inverting input)
5	V+ (non-inverting input)
6	-Vs (supply voltage)
7	no connection
8	no connection
9	offset control
10	output
11	+Vs (supply voltage)
12	no connection
13	no connection
14	

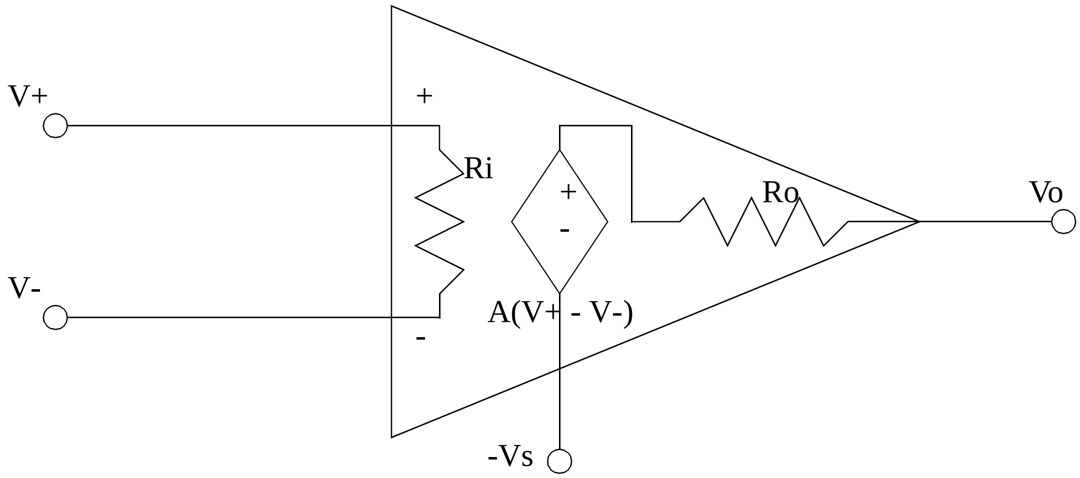
• The basic layout of the 741 op-amp is given below for the 8 pin dip package.



Pin #	Description
1	offset control
2	V- (inverting input)
3	V+ (non-inverting input)
4	-Vs (supply)
5	offset control
6	output
7	+Vs (supply)
8	no connection

5.1.3 Op-Amp Equivalent Circuits

• An equivalent circuit for an op-amp is given below,

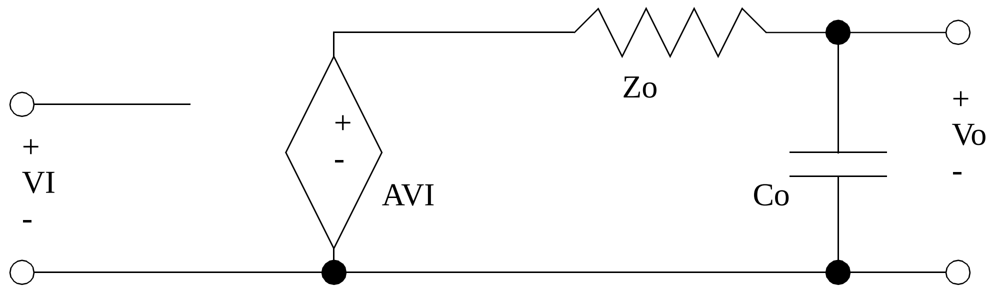


Where typical values are,

Ri = 1MΩ
Ro = 75W
A = 10^5

5.1.3.1 - Frequency Response

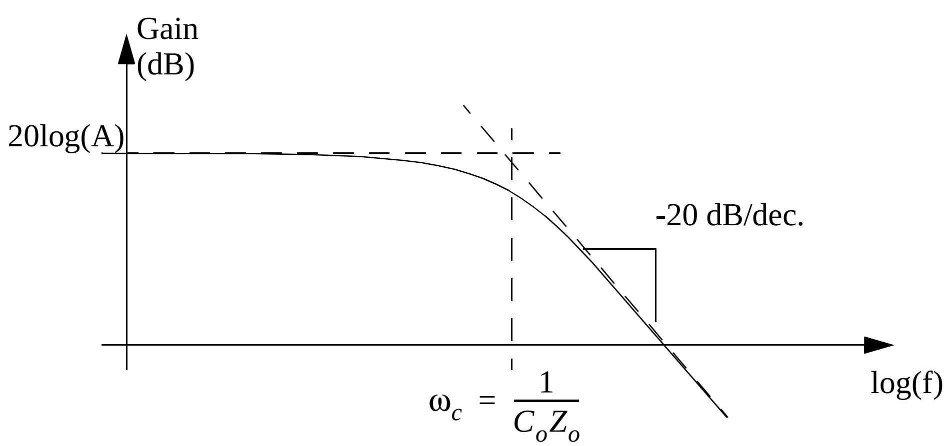
• Open loop frequency response can be estimated using the equivalent circuit below,



Basically this is a voltage divider. We can quickly find the gain,

$$V_o = (AV_I)\left(\frac{\frac{1}{j\omega C_o}}{Z_o + \frac{1}{j\omega C_o}}\right) = \frac{AV_I}{j\omega C_o Z_o + 1}$$

$$\boxed{\frac{V_o}{V_I} = \frac{A}{j\omega C_o Z_o + 1}} \qquad \text{Gain}$$

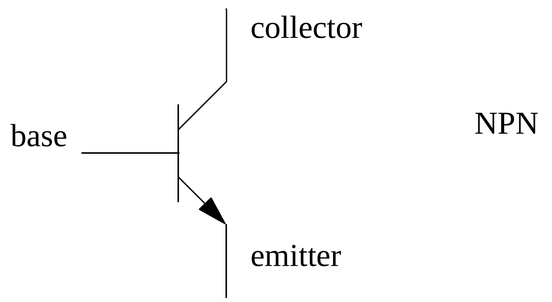


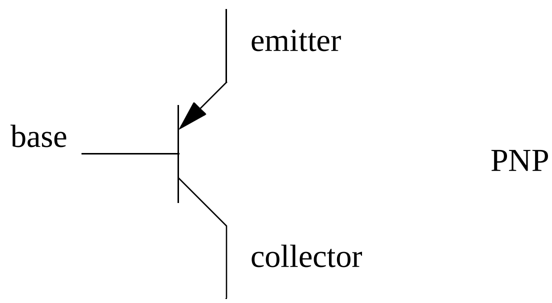
5.2 TRANSISTORS

5.2.1 Bipolar Junction Transistors (BJT)

• Bipolar Junction Transistors (BJTs) are made with three layers of doped silicon. The layers are either doped to be positive (p-type) or negative (n-type) using low concentrations of elements mixed with the silicon.

• There are two basic types, PNP and NPN. Their names come from the sequence of doped layers in the transistor. The schematic symbols for these transistors are shown below.





• The base-emitter voltage is ussualy given as a constant. This junction acts much like a diode, and will on average have voltages around 0.7V.

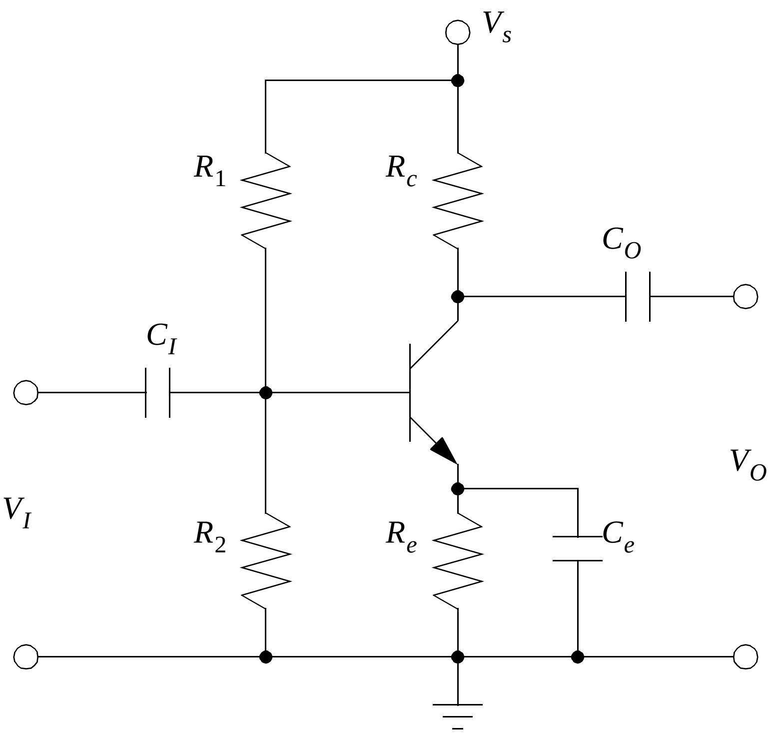
• Transistors are highly non-linear, but they are often biased by carefully applying voltages and currents to put them in a roughly linear range.

• A designer will depend heavily upon specifications. These are often in the form of graphs for different transistor applications.

• Except for applications such as switching, most transistor configurations are used for sinusoidal signals. As a result there is ussualy a DC design, as well as AC.

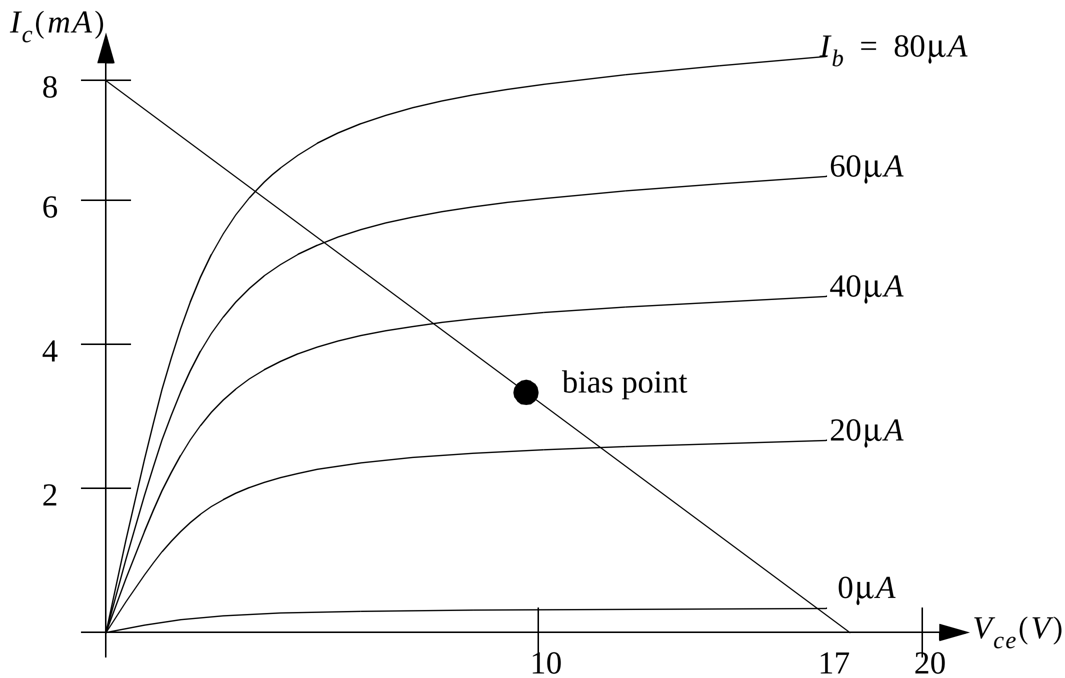
5.2.1.1 - Biasing Common Emitter Transistors

• A common emitter configuration is shown in the figure below.



• Consider the common emitter amplifier shown. The resistors provide DC biasing to select an operating point. The capacitor Ce is used to allow the AC to bypass Re.

• To perform the design we must first bias the transistor using the curves below.



$$R_c = \frac{1}{\text{slope of bias line}} = \frac{17V}{8mA} = 2125\Omega \approx 2.2K\Omega$$

a reasonable bias point is chosen to be near the center of the linear range, thus giving,

$$V_{ce} = 9V$$
$$I_e = 30\mu A$$
$$I_c = 4mA$$

Next we will assume the supply voltage is Vs = 20V. Based on collector current we can determine that

$$V_e = V_s - V_{ce} - I_c R_c = I_e R_e \qquad \text{and} \qquad I_e \approx \beta I_b$$

$$\therefore V_s - V_{ce} - I_c R_c = \beta I_b$$

Assuming we find a beta of 100 in the transistor specifications,

$$(20V) - (9V) - (4mA)(2200\Omega) = 100(30\mu A)R_e$$
$$R_e = 733\Omega \approx 720\Omega$$

Now, to select values for R1 and R2 we need to calculate Vb using Vbe from the specifications. (0.7V is a typical value).

$$V_b = V_e + V_{be} = I_e R_e + V_{be} = \beta I_b R_e + V_{be}$$
$$\therefore \quad = 100(30\mu A)720\Omega + 0.7V$$
$$\therefore \quad = 2.86V$$
$$V_b = V_s\left(\frac{R_2}{R_1 + R_2}\right)$$
$$2.86V = 20V\left(\frac{R_2}{R_1 + R_2}\right)$$
$$R_1 + R_2 = 7R_2$$
$$R_1 = 6R_2$$

The value of either R1 or R2 must be picked. To do this a common rule of thumb is to use a value of R2 that is approximate 10 time Re.

$$R_2 \approx 10R_e$$
$$R_2 = 7.2K\Omega$$
$$R_1 = 6(7.2K\Omega) = 43.2K\Omega \approx$$

6. AC CIRCUIT ANALYSIS

• There are a number of techniques used for analysing non-DC circuits.

• These techniques are,
- phasors - for single frequency, steady state systems
- laplace transforms - to find steady state as well as transient responses
- etc

6.1 PHASORS

• Phasors are used for the analysis of sinusoidal, steady state conditions.

• Sinusoidal means that if we measure the voltage (or current) at any point 'i' in the circuit it will have the general form,

$$V_i(t) = V_{i_{peak}} \sin(\omega_i t + \phi_i)$$

where,

i = a node number

$V_i(t)$ = the instantaneueous voltage at node i

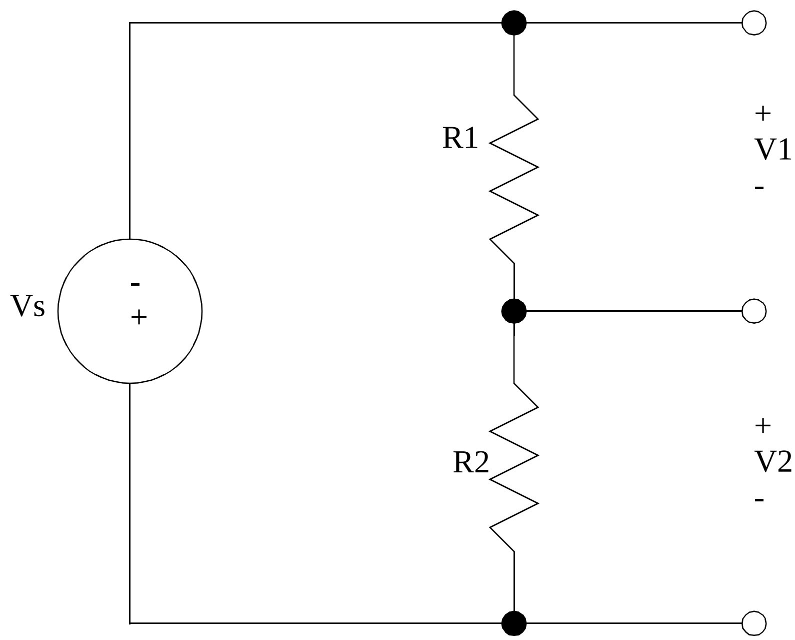
$V_{i_{peak}}$ = the peak voltage at node i

ω_i = the frequency of the sinusoid

ϕ_i = the phase shift

• Steady state means that the transients have all stopped. This can be crudely though of as the circuit has 'charged-up' or 'warmed-up'.

• Consider the example below,



Considering that this is a simple voltage divider,

$$V_2 = (-V_s)\left(\frac{R_2}{R_1 + R_2}\right)$$

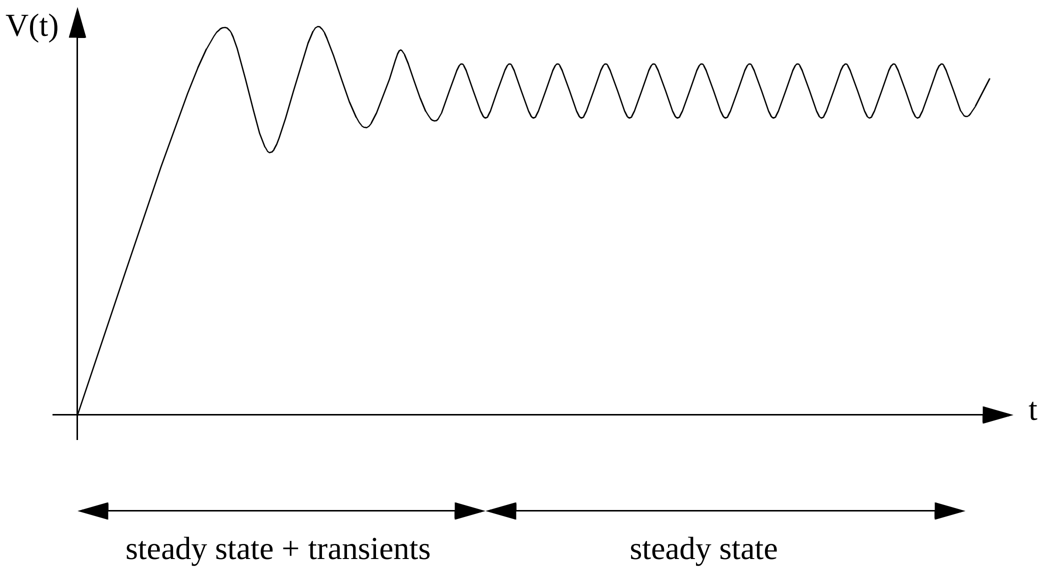
If the supply voltage is sinusoidal we would find,

$$V_s = 95\sin(10t + 0.3)$$

$$\therefore V_2 = (-95\sin(10t + 0.3))\left(\frac{R_2}{R_1 + R_2}\right)$$

$$\therefore V_2 = (95\sin(10t + 0.3 - \pi))\left(\frac{R_2}{R_1 + R_2}\right)$$

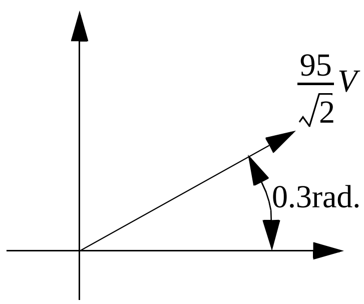
- Steady state is another important concept, it means that we are not concerned with the initial effects when we start a circuit (these effects are known as the transients). The typical causes of transient effects are inductors and capacitors.



• We typically deal with these problems using phasor analysis. In the example before we had a voltage represented in the time domain,

$$V_s = 95\sin(10t + 0.3)$$

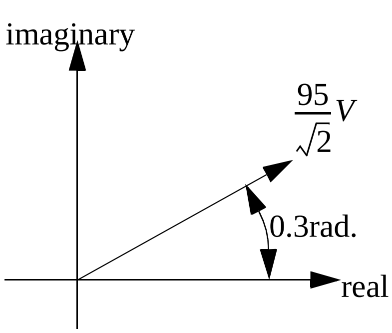
We could also represent this in the polar domain using magnitude and phase shift. These Phase diagrams are only applicable for a single frequency. Note: the peak value is divided by the square root of 2 to convert it to an RMS value.



OR

$$V_s = \frac{95}{\sqrt{2}} V \angle 0.3 rad$$

Finally, we could represent the values in complex form,

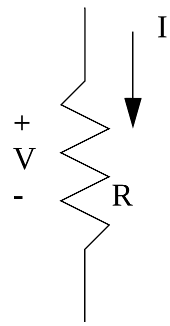
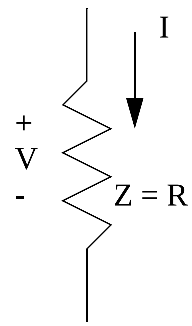
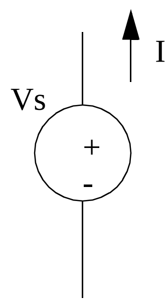
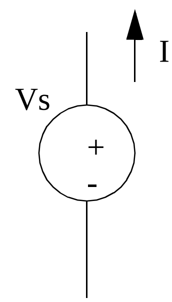
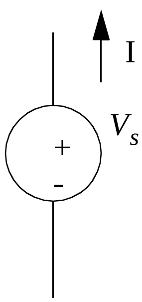
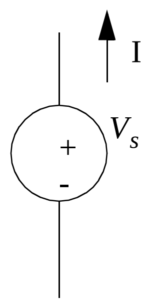
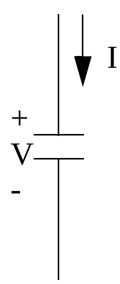
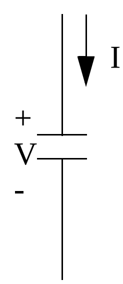


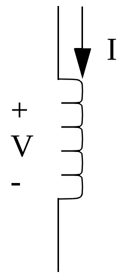
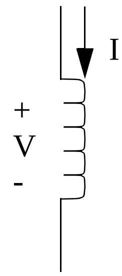
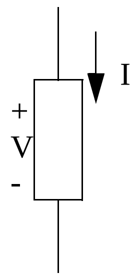
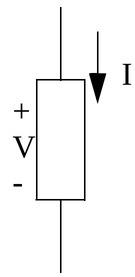
$$V_s = \frac{95}{\sqrt{2}} V\{e^{j\phi}\}$$

$$\therefore \quad = \frac{95}{\sqrt{2}} V\{\cos(0.3rad) + j\sin(0.3rad)\}$$

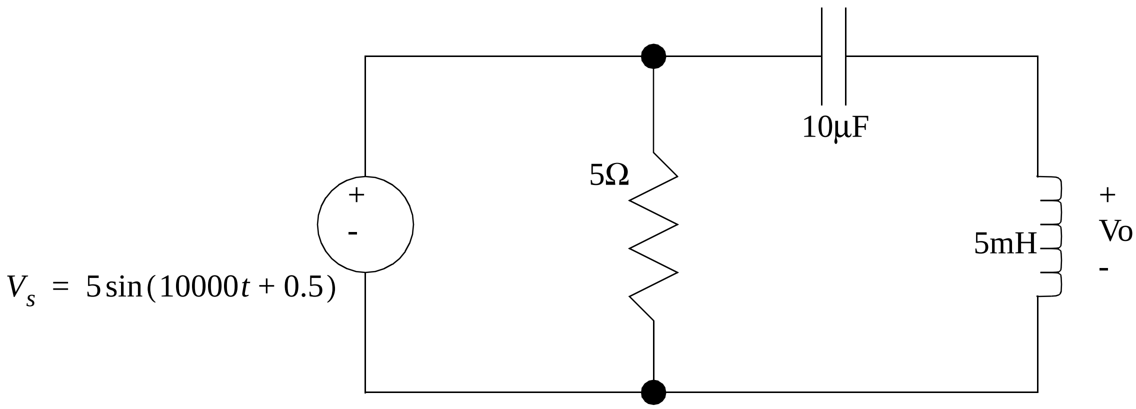
NOTE: When doing phasor analysis, it is assumed that all of the frequencies in the circuit are the same, and they only differ by a phase angle.

- Basically to do this type of analysis we represent all components voltages and currents in complex form, and then do calculations as normal.

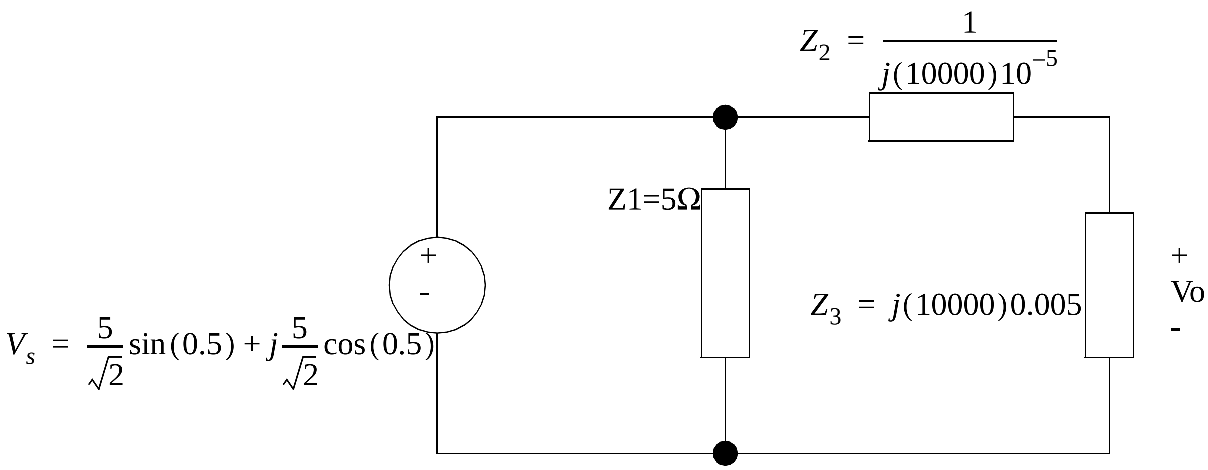
	TIME DOMAIN	PHASOR (FREQUENCY) DOMAIN
RESISTOR		
DC VOLTAGE SOURCE		
AC VOLTAGE SOURCE	 $V_s = A\sin(\omega t + \phi)$	 $V_s = \frac{A}{\sqrt{2}}\cos\phi + j\frac{A}{\sqrt{2}}\sin\phi$
CAPACITOR	 $C\frac{d}{dt}V = I$	 $Z = \frac{1}{j\omega C}$

	TIME DOMAIN		PHASOR (FREQUENCY) DOMAIN	
INDUCTOR		$V = L\frac{d}{dt}I$		$Z = j\omega L$
OHM'S LAW		$V = IR$		$V = IZ$

• Consider the simple example below,



We can redraw the diagram using impedances for each component,



Then, as if the circuit is only made of resistors, we procede to use standard circuit analysis techniques,

$$V_o = V_s\left(\frac{Z_3}{Z_2 + Z_3}\right) = \left(\frac{5}{\sqrt{2}}\sin(0.5) + j\frac{5}{\sqrt{2}}\cos(0.5)\right)\left(\frac{j(10000)0.005}{\frac{1}{j(10000)10^{-5}} + j(10000)0.005}\right)$$

$$\therefore V_o = (3.11 + j1.70)\left(\frac{(j(10000)0.005)(j(10000)10^{-5})}{1 + (j(10000)0.005)(j(10000)10^{-5})}\right)$$

$$\therefore V_o = (3.11 + j1.70)\left(\frac{5}{1-5}\right) = \{-3.89 - j2.13\}V = 4.43V\angle -2.07rad$$

If we express the output voltage as a function of time we get,

$$V_o(t) = 4.43\sin(10000t - 2.07)V$$

6.1.1 RMS Values

• When dealing with alternating currents we are faced with the problem of how we represent the signal magnitude. One easy way is to use the peak values for the wave.

• Another common method is to use the effective value. This is also known as the Root Mean Squared value.

$$V_{RMS} = \sqrt{\frac{1}{T}\int_0^T V^2(t)dt}$$

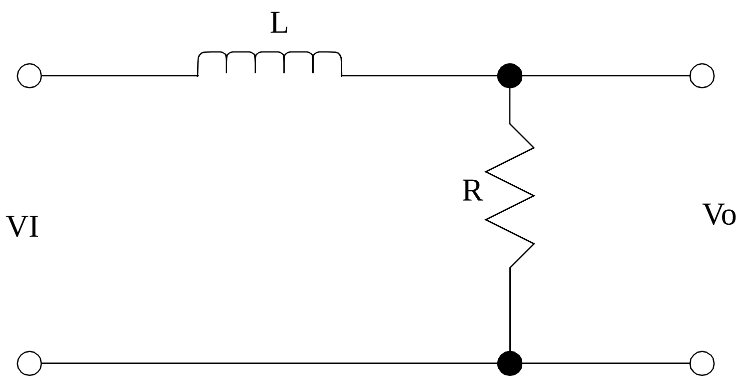
For a sinusoidal function we will find that,

$$V_{RMS} = \frac{V_{PEAK}}{\sqrt{2}} = 0.707V_{PEAK}$$

$$I_{RMS} = \frac{I_{PEAK}}{\sqrt{2}} = 0.707I_{PEAK}$$

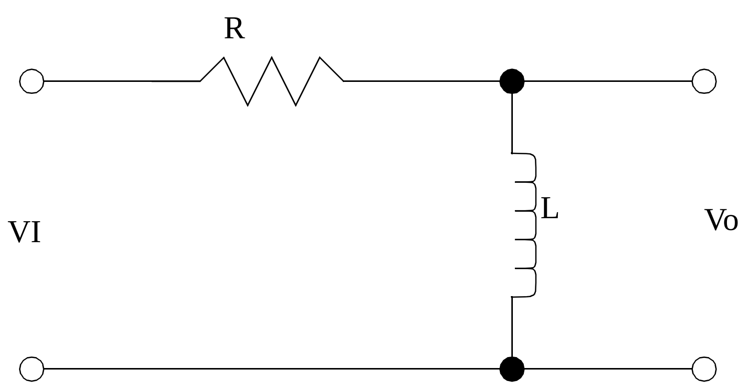
6.1.2 LR Circuits

• One common combination of components is an inductor and resistor.



(A low pass filter)

$$V_o = V_I\left(\frac{R}{R + jL\omega}\right)$$

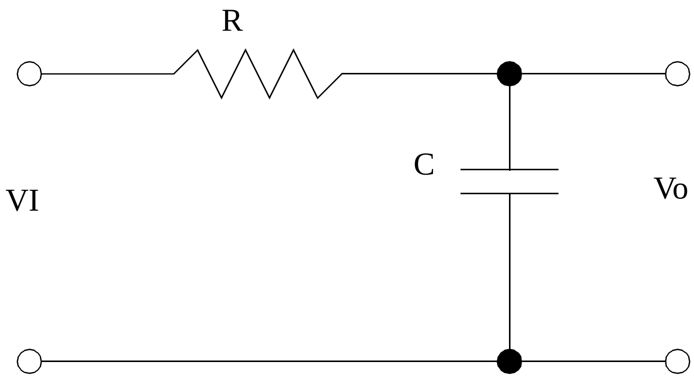


(A high pass filter)

$$V_o = V_I\left(\frac{jL\omega}{R + jL\omega}\right)$$

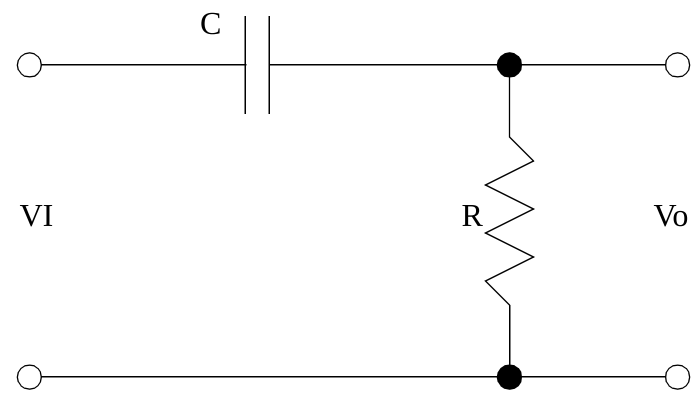
6.1.3 RC Circuits

• Capacitors are often teamed up with resistors to be used as filters,



(low pass filter)

$$V_o = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = \frac{1}{j\omega CR + 1}$$

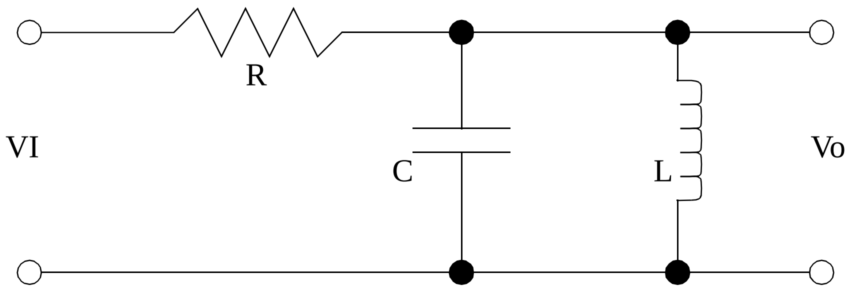


(high pass filter)

$$V_o = \frac{R}{R + \frac{1}{j\omega C}} = \frac{j\omega CR}{j\omega CR + 1}$$

6.1.4 LRC Circuits

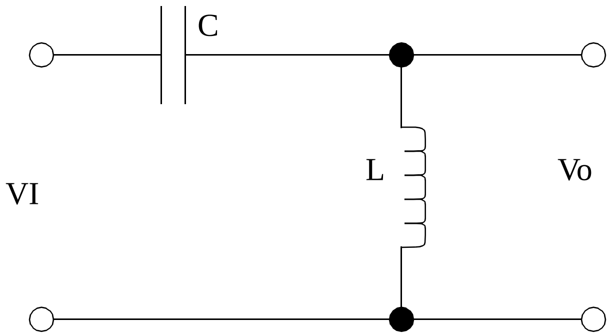
• These circuits tend to weigh off capacitors and inductors to have a preferred frequency.



$$V_o = V_I\left(\frac{\left(\frac{1}{\frac{1}{j\omega L} + j\omega C}\right)}{R + \left(\frac{1}{\frac{1}{j\omega L} + j\omega C}\right)}\right) = V_I\left(\frac{\left(\frac{j\omega L}{1 + (j\omega C)(j\omega L)}\right)}{R + \left(\frac{j\omega L}{1 + (j\omega C)(j\omega L)}\right)}\right) = \frac{j\omega L}{j\omega L + R(1 - \omega^2 LC)}$$

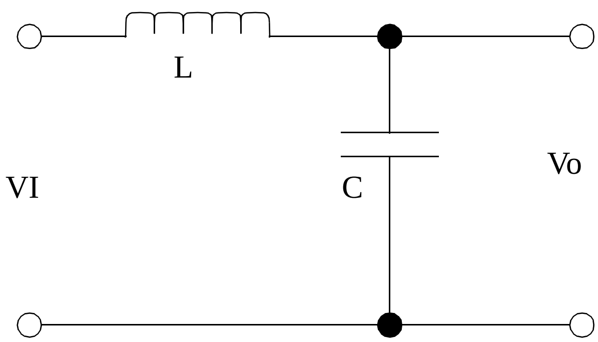
<u>6.1.5 LC Circuits</u>

• Inductor capacitor combinations can be useful when attempting to filter certain frequencies,



(high pass filter)

$$V_o = V_I\left(\frac{j\omega L}{j\omega L + \frac{1}{j\omega C}}\right) = V_I\left(\frac{\omega^2 LC}{\omega^2 LC - 1}\right)$$

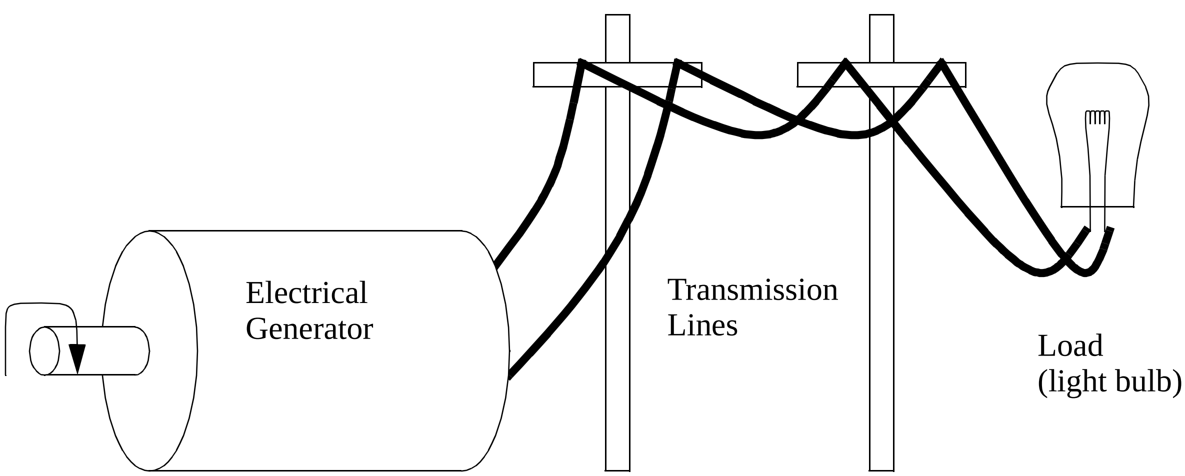


(low pass filter)

$$V_o = V_I\left(\frac{\frac{1}{j\omega C}}{j\omega L + \frac{1}{j\omega C}}\right) = V_I\left(\frac{1}{1 - \omega^2 LC}\right)$$

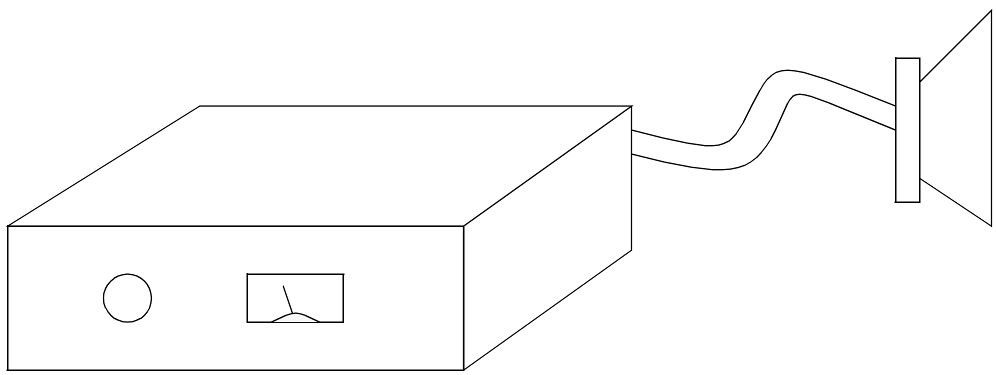
6.2 AC POWER

• Consider the power system shown below,



• The generator converts some form of mechanical force into electrical power. This power is then distributed to consumers over wires (and through transformers). Finally at the point of application, each load will draw a certain current, at the supply voltage - operating at a rated power. The voltages supplied this way are almost exclusively AC. Also in an ideal situation the load will be pure resistance, but in reality it will be somewhat reactive.

• Another important example of power delivered is when impedence matching between audio amplifiers and audio speakers. Most consumer systems are 50ohm for maximum power transfer and minimum distortion.



6.2.1 Complex Power

• Consider the basic power equation,

$$P = IV = I^2 Z = \frac{V^2}{Z}$$

if we consider the impedance and voltage in variable form,

$$Z = A + jB$$

$$V = C + jD$$

$$P = \frac{(C + jD)^2}{A + jB} = \frac{(C + jD)^2}{A + jB}\left(\frac{A - jB}{A - jB}\right)$$

$$\therefore P = \frac{(C^2 + 2jCD - D^2)(A - jB)}{A^2 + B^2}$$

$$\therefore P = \frac{((C^2 - D^2) + j2CD)(A - jB)}{A^2 + B^2}$$

$$\therefore P = \frac{[A(C^2 - D^2) - 2BCD] + j[2ACD + B(C^2 - D^2)]}{A^2 + B^2}$$

In a general form the results are,

$$S = P + jQ = P_{peak}\cos(2\omega t + \theta)$$

where,

S = the complex power
P = real power
Q = reactive power
theta = power angle

$$pf = \cos\theta = \frac{P}{\sqrt{P^2 + Q^2}}$$

where,

p.f. = power factor

6.2.1.1 - Real Power

- The relationship for real power is shown below where the current and resistance are in phase (although the values are rarely perfectly in phase).

$$P = IV = (V_p \sin\omega t)(I_p \sin\omega t) = V_p I_p (\sin\omega t)^2 = \frac{V_p I_p}{2}\sin(2\omega t) = V_{rms} I_{rms} \sin(2\omega t)$$

• When the current and voltage are D.C. (not charging) the circuit contains pure resistance, and the power is constantly dissipated as heat or otherwise. Notice that the value of P will always be positive, thus it never returns power to the circuit.

6.2.1.2 - Average Power

• An average power can be a good measure of real power consumption of a resistive component.

$$P = \frac{1}{T}\int_0^T p\,dt$$

Consider the case of the pure resistance,

$$P = \frac{V_p I_p}{2\pi}\int_0^{2\pi} (\sin t)^2 dt$$

6.2.1.3 - Reactive Power

• When we have a circuit component that has current ±90° out of phase with the voltage it uses reactive power. In this case the net power consumption is zero, in actuality the power is stored in and released from magnetic or electric fields.

• Consider the following calculations,

$$p = (V_p \sin\omega t)\left(I_p \sin\left(\omega t + \frac{\pi}{2}\right)\right)$$

$$\therefore p = V_p I_p (\sin\omega t)\left(\sin\left(\omega t + \frac{\pi}{2}\right)\right)$$

$$\therefore p = V_p I_p (\sin\omega t)(\cos\omega t)$$

$$\therefore p = V_p I_p \left(\frac{\sin 2\omega t}{2}\right) = V_{rms} I_{rms} \sin(2\omega t)$$

Here the power will be positive then negative. In this case if we integrate power consumption over a cycle there is no net comsumption.

6.2.1.4 - Apparent Power

• In all circuits we have some combination of Real and Reactive power. We can combine these into one quantity called apparent power,

$$S = P + jQ$$

where,
S = apparent power (VA)
P = real power from voltage and current in phase (W)
Q = reactive power from voltage and current 90° out of phase (var)

6.2.1.5 - Complex Power

• We can continue the examination of power by assuming each is as below,

$$S = VI^*$$

$$\therefore \quad = (Ve^{j\theta_v})(Ie^{-j(\theta_v - \theta_I)}) = VIe^{j\theta_I} = VI(\cos\theta_I + j\sin\theta_I)$$

6.2.1.6 - Power Factor

• The power factor (p.f.) is a good measure of how well a power source is being used.

$$pf = \frac{P}{|S|} = \frac{P}{\sqrt{P^2 + Q^2}}$$

As this value approaches 1 the power consumption becomes entirely real, and the load becomes purely resistive.

• It is common to try to correct power factor values when in industrial settings. For example, if a large motor were connected to a power grid, it would introduce an inductive effect. Capacitors can be added to compensate.

6.2.1.7 - Average Power Calculation

• If we want to find the average power, consider the following,

$$P_{avg} = \frac{1}{T}\int_0^T I^2(t)Rdt$$

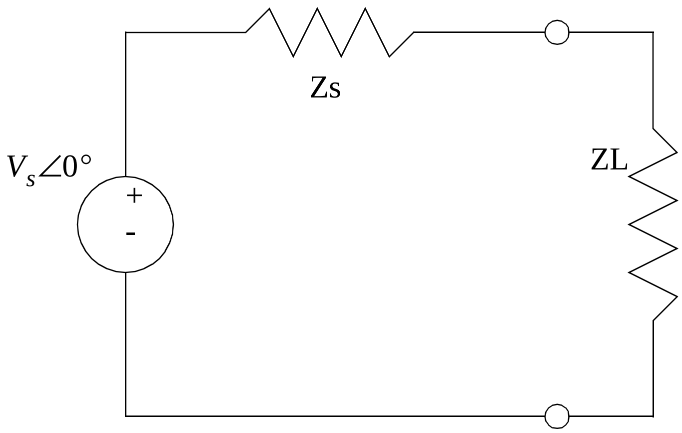
$$\therefore \quad = R\left(\frac{1}{T}\int_0^T I^2(t)dt\right) = RI_{RMS}^2$$

Likewise,

$$P_{avg} = \frac{V_{RMS}^2}{R} = I_{RMS}V_{RMS}$$

6.2.1.8 - Maximum Power Transfer

• Consider the thevenin circuit below. We want to find the maximum power transfered from this circuit to the external resistance.



$$Z_s = R_s + jX_s$$

$$Z_L = R_L + jX_L$$

$$I = \frac{V_s\angle 0°}{Z_s + Z_L} = \frac{V_s}{(R_s + R_L) + j(X_s + X_L)}$$

The average power delivered to the load is,

$$P_L = Z_L|I|^2$$

$$\therefore \quad = (R_L + jX_L)\left(\frac{V_s}{\sqrt{(R_s + R_L)^2 + j(X_s + X_L)^2}}\right)^2$$

$$\therefore \quad = \frac{V_s^2(R_L + jX_L)}{(R_s + R_L)^2 + (X_s + X_L)^2}$$

To find the maximum we can then take a partial derivative for the resistance,

$$\frac{\partial}{\partial R_L}P_L = \frac{V_s^2((R_s + R_L)^2 + (X_s + X_L)^2 - 2R_L(R_s + R_L))}{((R_s + R_L)^2 + (X_s + X_L)^2)^2} = 0$$

$$\therefore 0 = (R_s + R_L)^2 + (X_s + X_L)^2 - 2R_L(R_s + R_L)$$

$$\therefore 0 = R_L^2 + R_s^2 + 2R_LR_s + X_L^2 + X_s^2 + 2X_LX_s - 2R_L^2 - 2R_LR_s$$

$$\therefore R_L^2(-1) + R_L(0) + (R_s^2 + X_L^2 + X_s^2 + 2X_LX_s) = 0$$

$$\boxed{\therefore R_L = \sqrt{R_s^2 + (X_L + X_s)^2}}$$

Next, consider the partial derivative for the reactance,

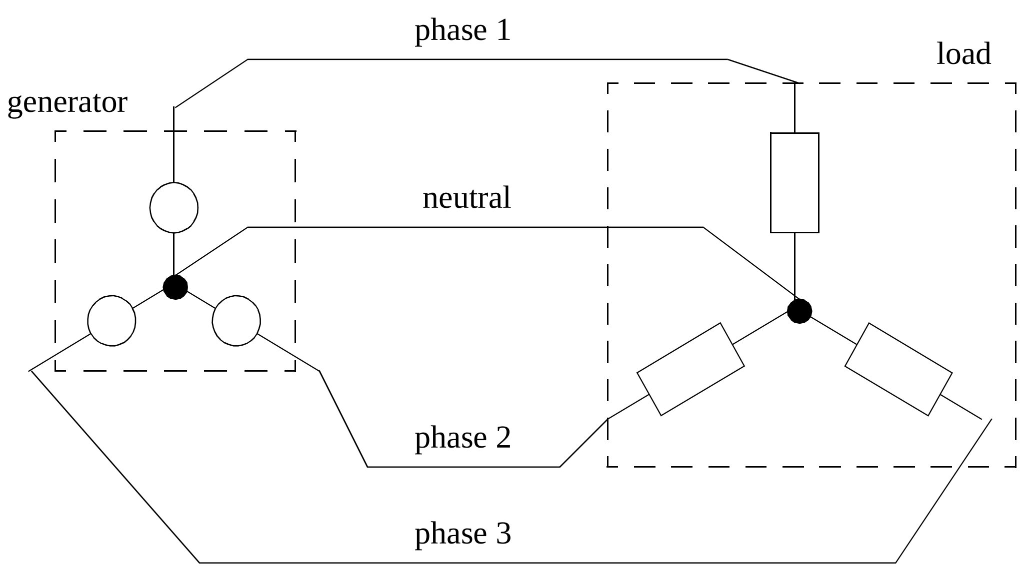
$$\frac{\partial}{\partial X_L} P_L = \frac{-V_s^2 2R_L (X_L + X_s)}{((R_L + R_s)^2 + (X_L + X_s)^2)^2} = 0$$

$$\boxed{\therefore X_L = -X_s}$$

6.3 3-PHASE CIRCUITS

• 3-phase circuits are common in large scale power generators and delivery systems.

• These systems carry 3 phases of voltage, each 120 degrees out of phase, on three separate conductors. If these three wires are connected through a balanced load the sum of currents is zero. Most systems provide a fourth wire as a neutral.

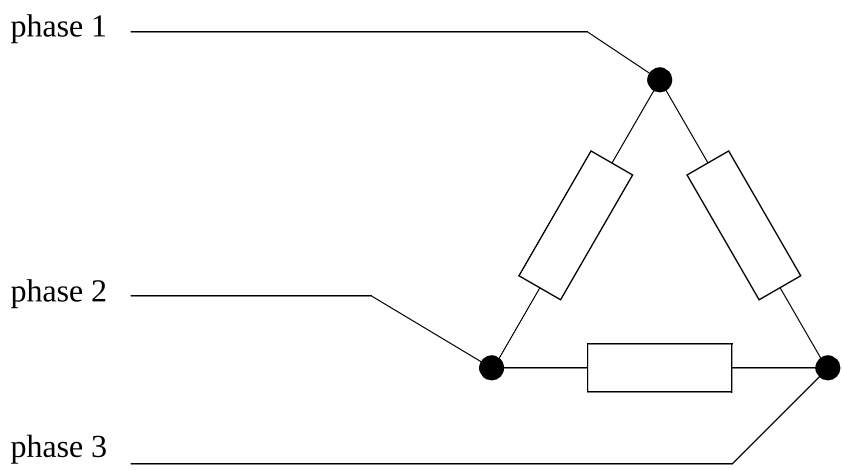


$$V_1(t) = V_{peak} \sin(\omega t) = V_{rms} \angle 0°$$

$$V_2(t) = V_{peak} \sin\left(\omega t + \frac{2\pi}{3}\right) = V_{rms} \angle 120°$$

$$V_3(t) = V_{peak} \sin\left(\omega t + \frac{4\pi}{3}\right) = V_{rms} \angle 240°$$

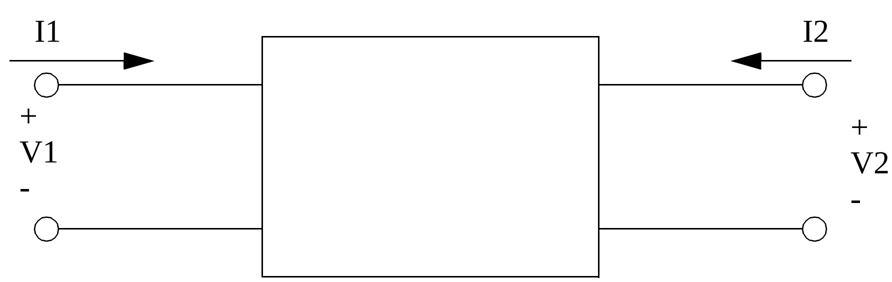
• As a result loads can be connected in a delta configuration with no neutral.



7. TWO PORT NETWORKS

• Two port networks are a useful tool for describing idealized components. The basic device schematics are simple, but each set of parameters views the device differently.

• The basic device is seen below, along with the various parameter sets,



z-parameters
$$V_1 = z_{11}I_1 + z_{12}I_2$$
$$V_2 = z_{21}I_1 + z_{22}I_2$$

inverse $\downarrow$

y-parameters
$$I_1 = y_{11}V_1 + y_{12}V_2$$
$$I_2 = y_{21}V_1 + y_{22}V_2$$

a-parameters
$$V_1 = a_{11}V_2 + a_{12}I_2$$
$$I_1 = a_{21}V_2 + a_{22}I_2$$

inverse $\downarrow$

b-parameters
$$V_2 = b_{11}V_1 + b_{12}I_1$$
$$I_2 = b_{21}V_1 + b_{22}I_1$$

h-parameters
$$V_1 = h_{11}I_1 + h_{12}V_2$$
$$I_2 = h_{21}I_1 + h_{22}V_2$$

inverse $\downarrow$

g-parameters
$$I_1 = g_{11}V_1 + g_{12}I_2$$
$$V_2 = g_{21}V_1 + g_{22}I_2$$

7.1 PARAMETER VALUES

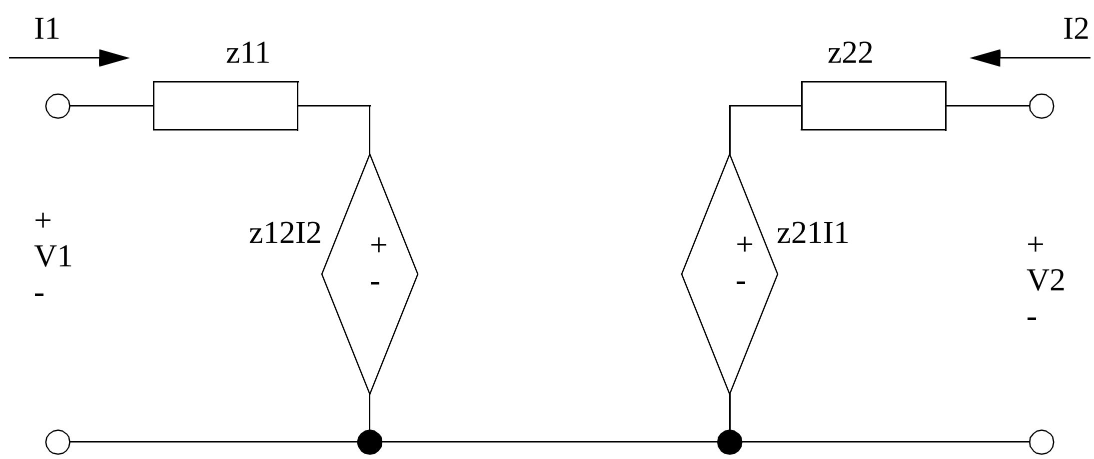
• obviously some of the parameters are impedance, while others are admittance. They can be easily determined by setting other parameters to zero, and measuring relevant voltages/currents.

7.1.1 z-Parameters (impedance)

• The values are as below,

parameter	units	description
$z_{11} = \left.\frac{V_1}{I_1}\right\rvert_{I_2 = 0}$	Ω	Input Impedance - port 1 impedance with port 2 open circuit.
$z_{12} = \left.\frac{V_1}{I_2}\right\rvert_{I_1 = 0}$	Ω	Transfer Impedance - ratio of port 1 voltage to port 2 current with port 1 open circuit
$z_{21} = \left.\frac{V_2}{I_1}\right\rvert_{I_2 = 0}$	Ω	Transfer Impedance - ratio of port 2 voltage to port 1 current with port 2 open circuit
$z_{22} = \left.\frac{V_2}{I_2}\right\rvert_{I_1 = 0}$	Ω	Output Impedance - the impedance of the output terminals with port 1 open circuit.

• The equivalent circuit for the z-parameters is shown below,



7.1.2 y-Parameters (admittance)

7.1.3 a-Parameters (transmission)

7.1.4 b-Parameters (transmission)

7.1.5 h-Parameters (hybrid)

7.1.6 g- Parameters (hybrid)

7.2 PROPERTIES

- If one set of parameters is known, other parameters can be found using simple conversions. This can help when one set of parameters is needed, but cannot be measured directly.

- Simple cases of networks are reciprocal and symetrical. When a network is neither of these, then it typically has active components, dependant sources, etc.

7.2.1 Reciprocal Networks

- If a voltage is applied at one port, the short circuit current out the other port will be the same, regardless of which side the voltage is applied to.

- Reciprocal networks are only possible when passive elements are used.

- The parameters that indicate a reciprocal networks are,

$$z_{12} = z_{21}$$

$$y_{12} = y_{21}$$

$$det\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = 1$$

$$det\begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = 1$$

$$h_{12} = -h_{21}$$

$$g_{12} = -g_{21}$$

• With any reciprocal network we only need to find 3 of the four parameters, the last can be determined mathematically.

7.2.2 Symmetrical Networks

• This is a special case of the reciprocal network where the input and output parameters are identical.

• In addition to the reciprocal constraints, we must also consider,

$$z_{11} = z_{22}$$

$$y_{11} = y_{22}$$

$$a_{11} = a_{22}$$

$$b_{11} = b_{22}$$

$$det\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = 1$$

$$det\begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} = 1$$

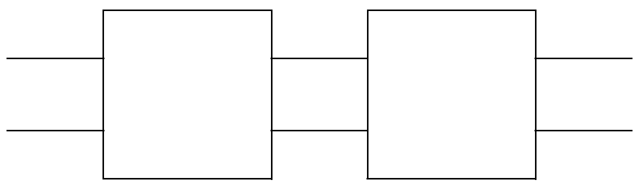
• Only two of these parameters need to be found to find the other two parameters.

7.3 CONNECTING NETWORKS

• When connecting networks, various parameters add ease.

7.3.1 Cascade

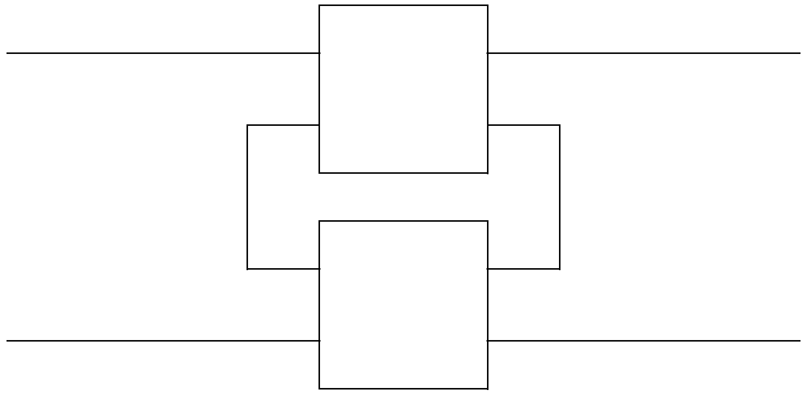
• The a parameters can be multiplied



$$a_{eq} = a_1 a_2$$

7.3.2 Series

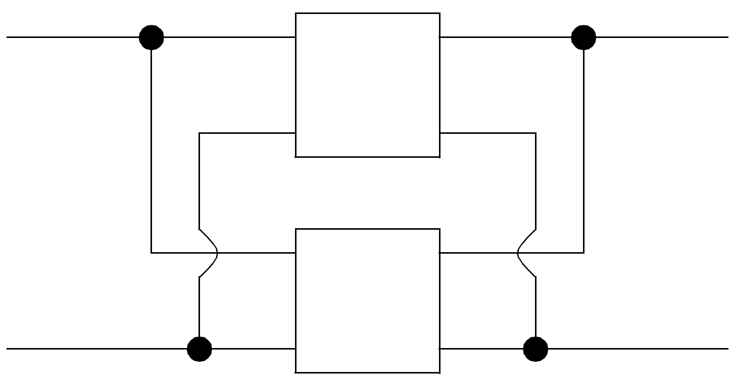
• With this type of connection the parameters are added,



$$z_{eq} = z_1 + z_2$$

7.3.3 Parallel

• Here the devices are connected,



7.3.4 Series-Parallel

7.3.5 Parallel-Series

8. CAE TECHNIQUES FOR CIRCUITS

9. A CIRCUITS COOKBOOK

9.1 HOW TO USE A COOKBOOK

• A cookbook is intended to provide enough examples of useful circuits to fill in black boxes in designs.

• Before using this section, the designer should already have some concept of what they want their circuit to do, and have a block diagram of function. In this section you can find ways to fill the black boxes.

9.2 SAFETY

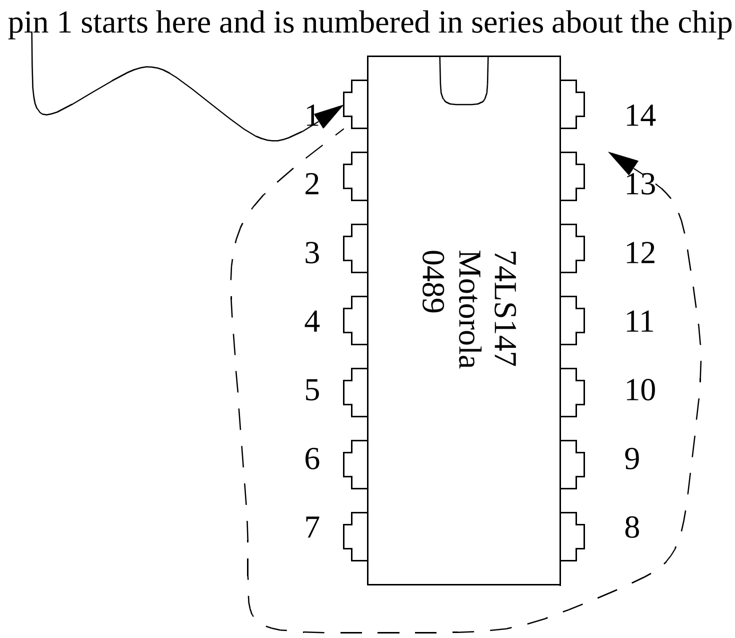
• Although you may be familiar with safety, it is always worth a review. A few careless minutes in a lab can be fatal.

• Some (BUT NOT ALL) safety rules are,

1. Always think about what you are doing, don't try something without understanding the consequences.
2. Cover or insulate live contact points when not testing, and use insulated probes.
3. Keep objects shielded and properly grounded, and avoid ground loops.
4. Wet surfaces can make you a convenient path for electricity
5. When soldering remember the molten solder is hotter than boiling water, and can splatter/spray/etc. - wear safety glasses.
6. Remember electrolytic capacitors WILL EXPLODE IF CONNECTED BACKWARDS.
7. Use fuses when possible.
8. Double check, and look for stupid mistakes before turning a device on. Common problems are reversed power supply polarity, short circuits, loose connections.
9. Keep things clean while working , and leave the lab better than you found it.
10. If something has malfunctioned deal with it - report it, fix it, or throw it out.

9.3 BASIC NOTES ABOUT CHIPS

• The cases come in many forms, but for inhouse development the DIP (Dual In-Line Pin) package is most popular, and most chips here are numbered with the same pin convention, unless specified.



• Chips are labelled with part numbers, for example the 74F147, will logically be equivalent to the 74LS147, except that they will have different rated speeds. The ‘F’ signifies fast, and ‘LS’ signifies low speed.

• There are extensive volumes of databooks available for chips, these are typically low cost, and available at any vendor of microchips.

• Many manufacturers make common chips, with the same IC numbers. But, there are also many proprietary chips. Be wary when selecting a non-standard IC, small purchases may be frowned upon by the supplier, making them hard to get in quantities of less than 1000.

• Some IC manufacturers are,
  - Motorola
  - National Semiconductor
  - Texas Instruments
  -

• CMOS chips will need pull-up resistors on inputs.

• When TTL inputs have nothing attached they tend to “float high” and will indicate that an input is true.

9.4 CONVENTIONS

9.5 USEFUL COMPONENT INFORMATION

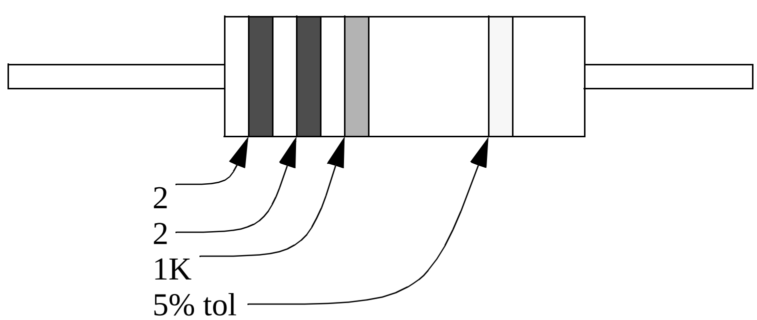
• There are basic families of standard components to be found. Many of these are marked by terse codes and symbols.

9.5.1 Resistors

• The resistor color code is,

Black = 0 = 1X
Brown = 1 = 10X
Red = 2 = 100X
Orange = 3 = 1000X
Yellow = 4 = 10000X
Green = 5 = 100,000X
Blue = 6 = 1,000,000X
Violet = 7 = 10,000,000X
Grey = 8 = 100,000,000X
White = 9 = 1,000,000,000X
Silver = 10% tolerance
Gold = 5% tolerance
Brown = 1% tolerance

• A resistor will have 4 or 5 bands. The bands that are grouped, or closer to one side of the resistor are the nominal value of the resistor. A single band will be set apart, this will be the tolerance.



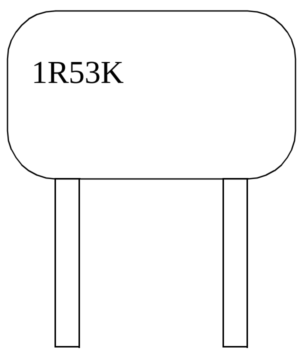
$$\therefore 22K\Omega \pm 5\%$$

9.5.2 Capacitors

• Capacitors quite often have values printed on them. When the values are not clear, there may be a number code used.

• Consider the capacitor with a number code below,

e.g.,



1	digit ‘1’
R	decimal place (optional)
5	digit ‘5’
3	multiplier x1000
K	tolerance ±10%

$\therefore 1.5\mu F \pm 10\%$

Multipliers

0 = x1
1 = x10
2 = x100
3 = x1000
4 = x10000
5 = x100000
8 = x0.01
9 = x0.1

Tolerances

B = ±0.1pF (<10pF)
C = ±0.25pF (<10pF)
D = ±0.5pF (<10pF)
F = ±1pF (<10pF) = ±1% (>10pF)
G = ±2pF (<10pF) = ±2% (>10pF)
H = ±3pF (>10pF)
J = ±5pF (>10pF)
K = ±10pF (>10pF)
M = ±20pF (>10pF)

9.6 FABRICATION

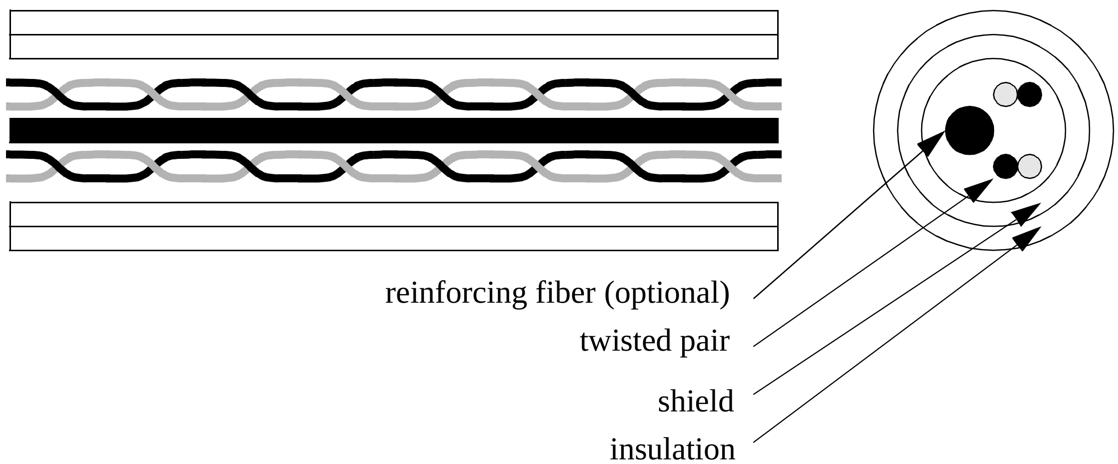
• There are a few popular methods of fabrication
 - wire wrap
 - bin board
 - bread board
 - circuit board

9.6.1 Shielding and Grounding

• Shielding is important for all circuits, it prevents electrical noise from creating false digital signals, and from corrupting analog signals.

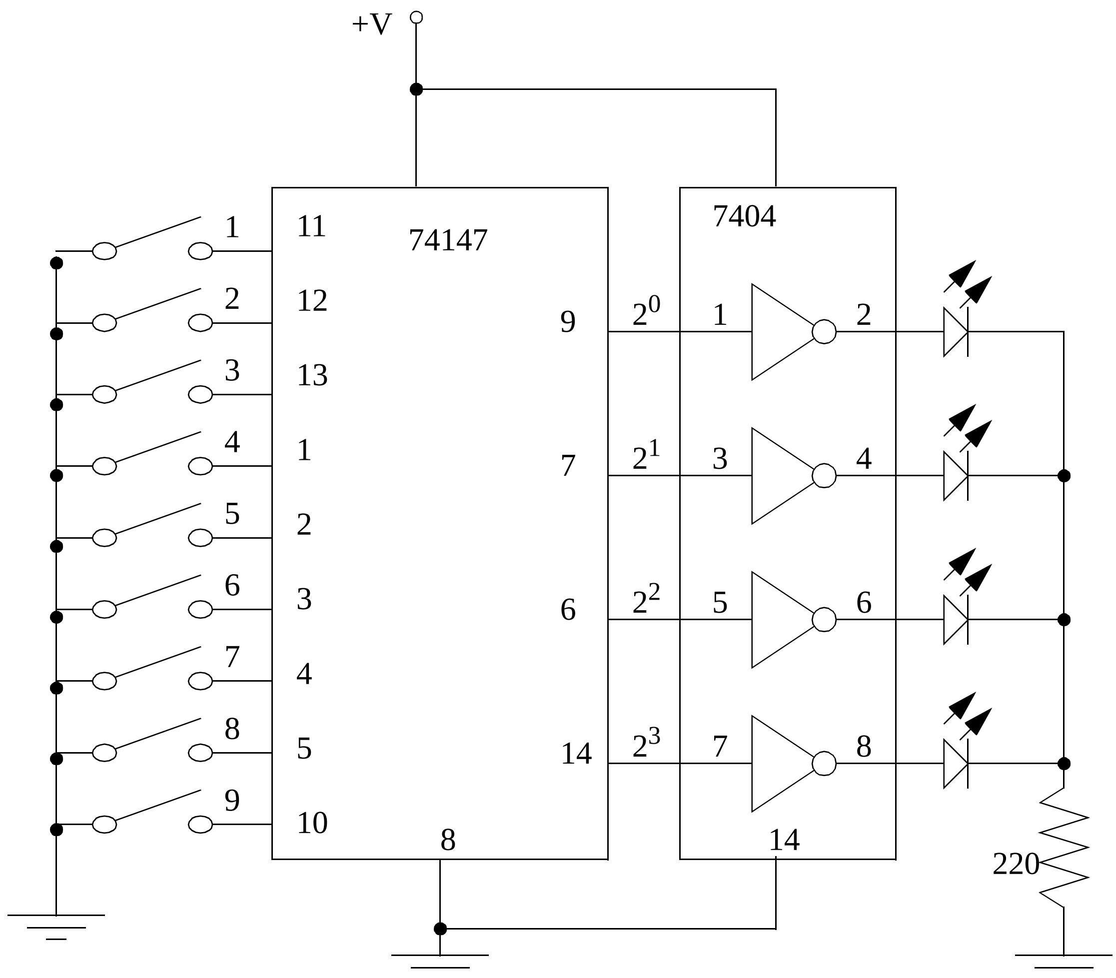
• Shielding is accomplished through a number of methods:
  - sheet metal (iron) enclosures keep electromagnetic interference out, or in.
  - shielded cables
  - RF chokes
  - bypass capacitors

• Cables can be shielded two different ways:
  - twisted pairs - two wires that are used for a signal (signal and common) are twisted once per inch or more. As a result, any inductive magnetic field induces a current one way for one twist, and the other way for the next twist - hence cancelling out the induced current.
  - shielding sheaths - cable bundles are often covered by a metal foil, or braided wire to provide a general protection for the cable. This shield is to be connected at one end (not two) of the cable to drain off any induced currents.

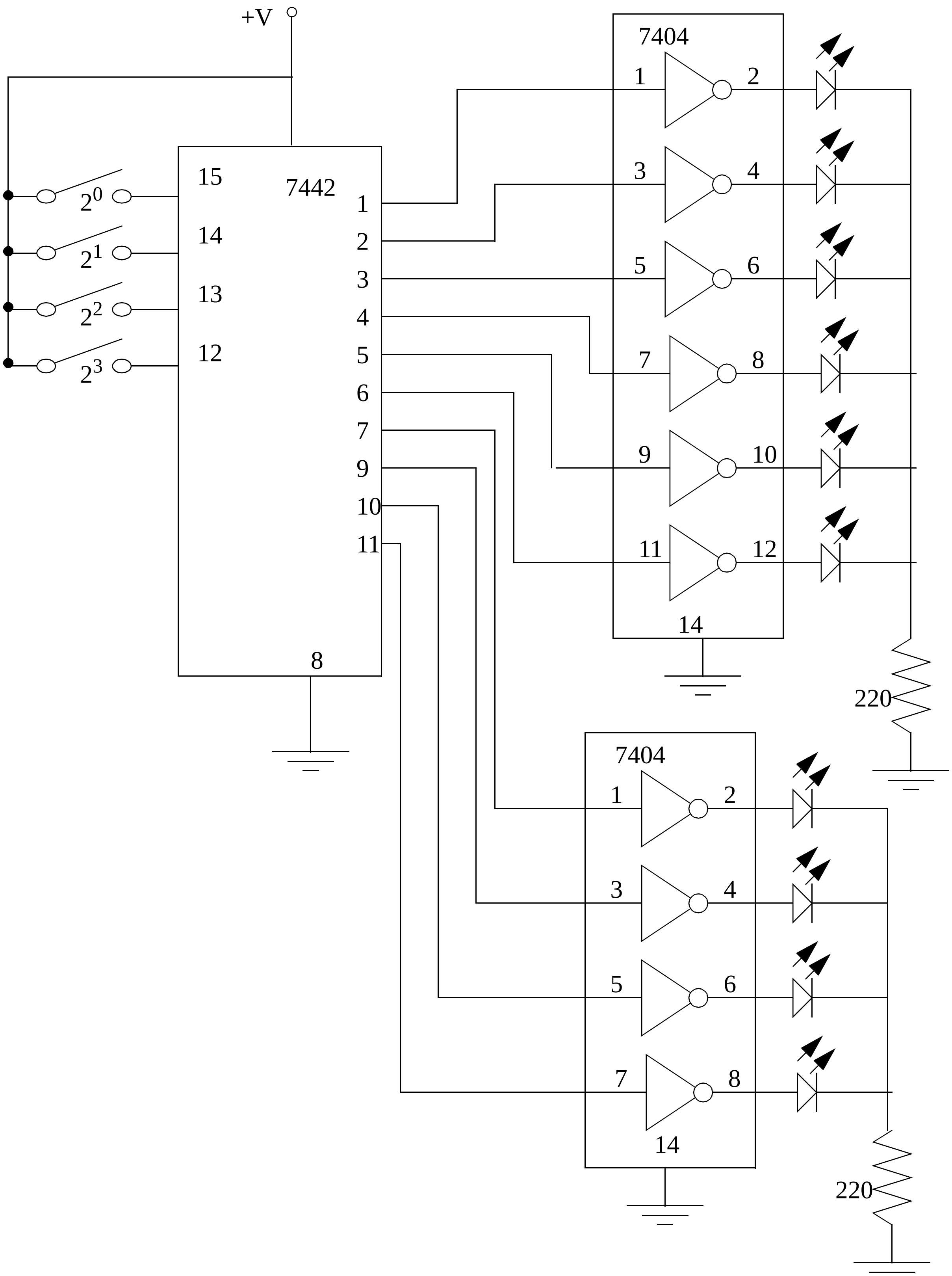


9.7 LOGIC

• Decimal to binary encoder

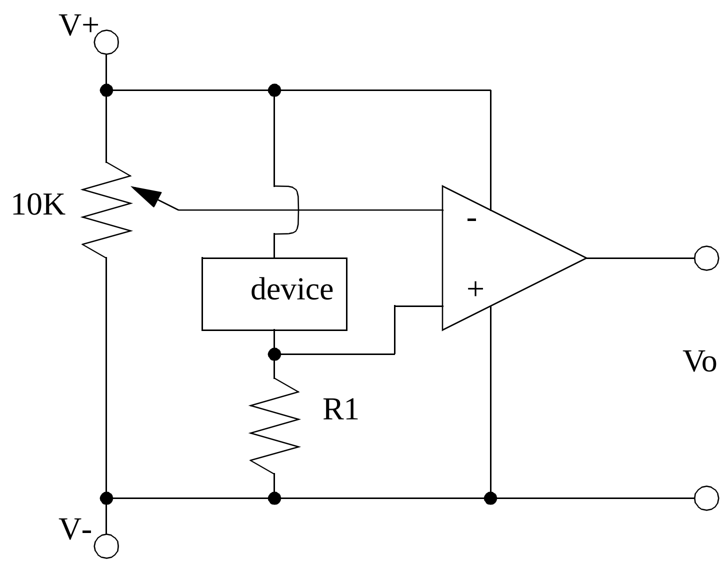


• Binary to decimal decoder

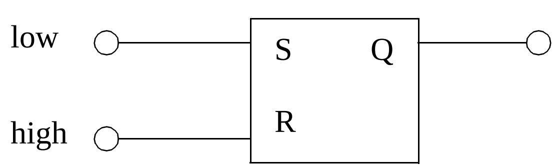

+V
7442
15
14
13
12
2^0
2^1
2^2
2^3
1
2
3
4
5
6
7
9
10
11
8
7404
1 2
3 4
5 6
7 8
9 10
11 12
14
220
7404
1 2
3 4
5 6
7 8
14
220

9.8 ANALOG SENSORS

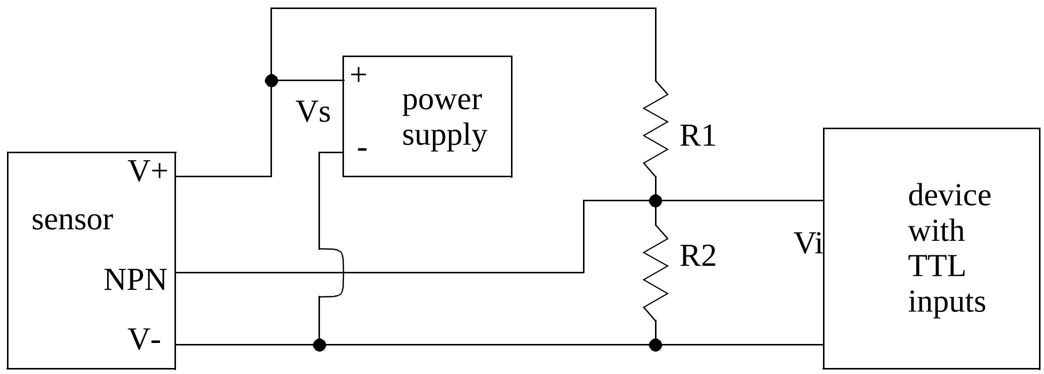
• LEVEL DETECTOR LIGHT OR TEMPERATURE - To measure temperatures or light levels against one level. If measuring temperature the device should be an RTD. If measuring light the device should be a photoresistor (LDR). The value of resistor R1 should be selected to be close to the normal resistance of the device. The potentiometer can be used to make fine adjustments.



• RANGE CONTROLLER - Upper/lower range controller. This can be done with a simple a simple flip flop. Two level detector circuits are used for the inputs. The Set value should be the upper range, the reset value should be the lower value. The output can be used to drive a relay, or some other driver.

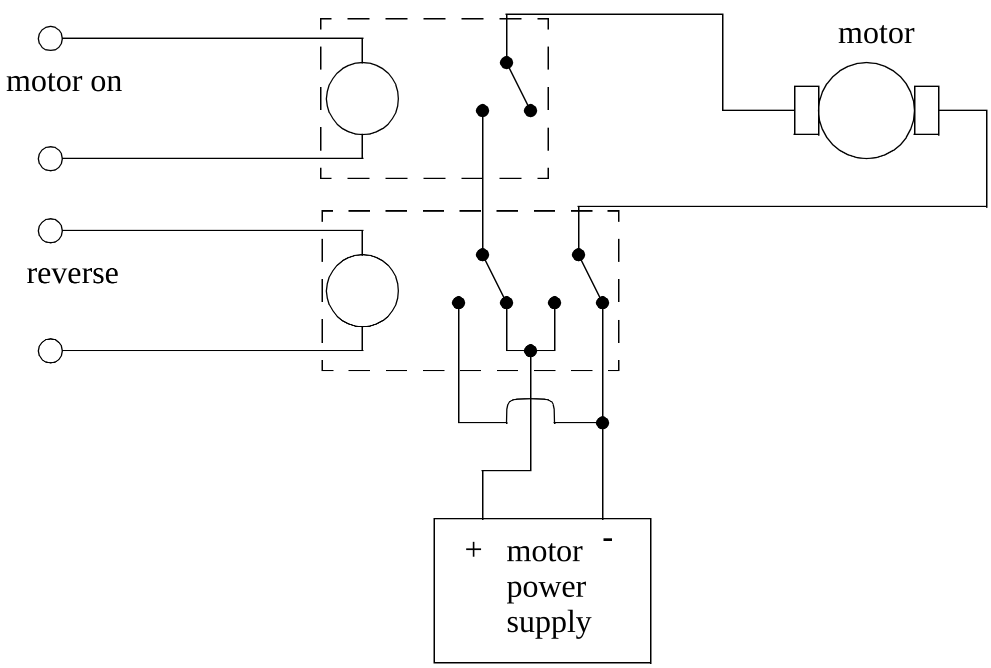


• SINKING SENSOR TO TTL - To convert a sinking sensor to a TTL input. The ratio of resistors R1 and R2 is determined by the ratio between the sensor supply voltage (normally 24V) and the TTL input voltage (normally 5V). The resisto values should probably be between 1K and 10K.

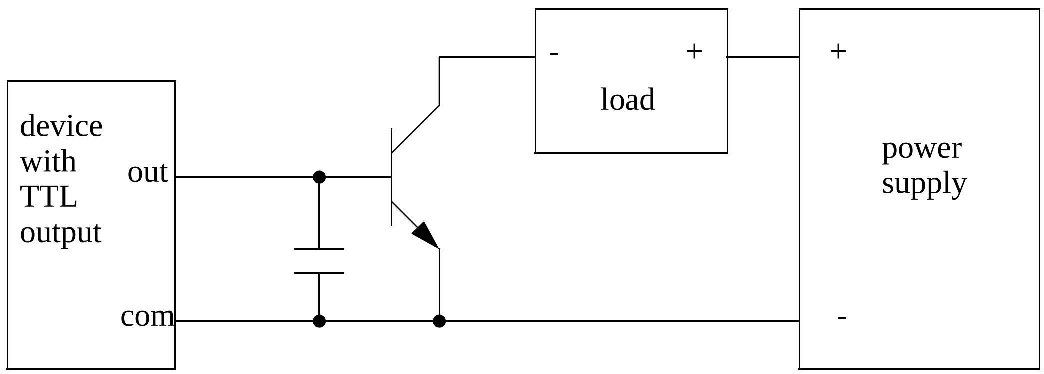


$$\frac{V_i}{V_s} = \frac{R_2}{R_1 + R_2}$$

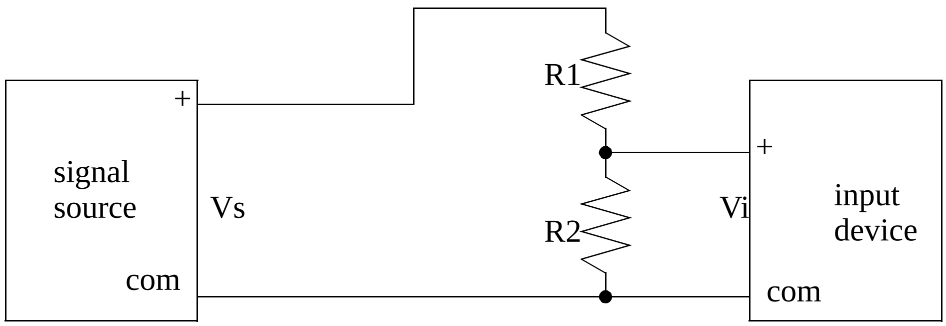
• MOTOR REVERSER USING RELAYS - A circuit that allows a motor to be turned on in either direction (safely). The motor on relay can be a single pole single throw (SPST), whilew the reversal relay mst be a double pole double throw (DPDT) relay. The relays should be selected to carry the peak motor currents.



• DRIVING A HIGH CURRENT DC LOAD WITH A TRANSISTOR - This circuit can be used for a load that requires a few amps of power, but is being controlled by a low current TTL output. The transistor must be selected so that it can carry the maximum load current. A heat sink should be used if the device will pass a large percentage of the rated current. Note that the voltage loss accross the transistor will be approximately 2V. For a higher current load a Darlington coupled transistor can be used.

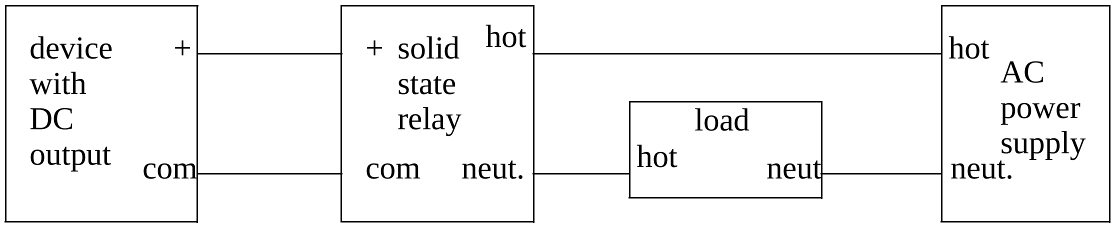


• SIGNAL VOLTAGE LEVEL REDUCTION - A higher voltage signal can be divided to a lower fraction using a voltage divider. This is only suitable for devices with high impedance inputs and should not be used to reduce battery voltages for motors, or other similar applications. The values of R1 and R2 should probably be about 10K.

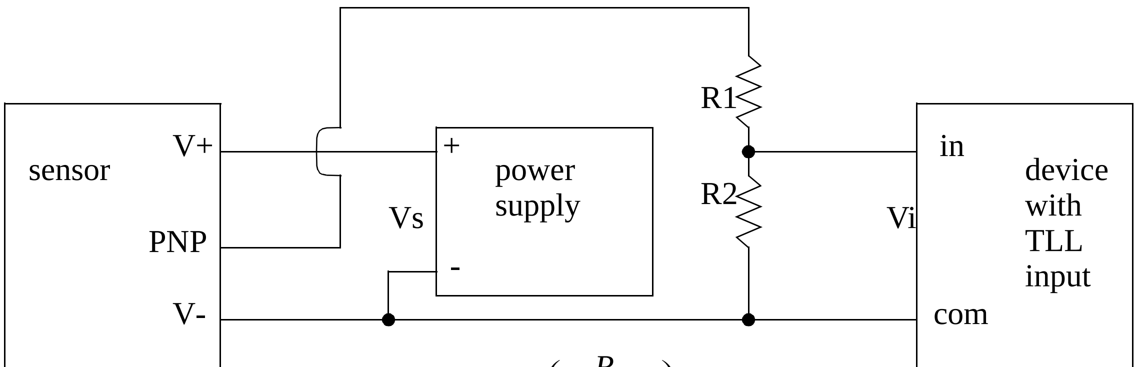


$$V_i = V_s\left(\frac{R_2}{R_1 + R_2}\right)$$

• SWITCHING AN AC LOAD WITH A SOLID STATE RELAY - AC loads can be controlled with a low current DC output using a solid state relay.

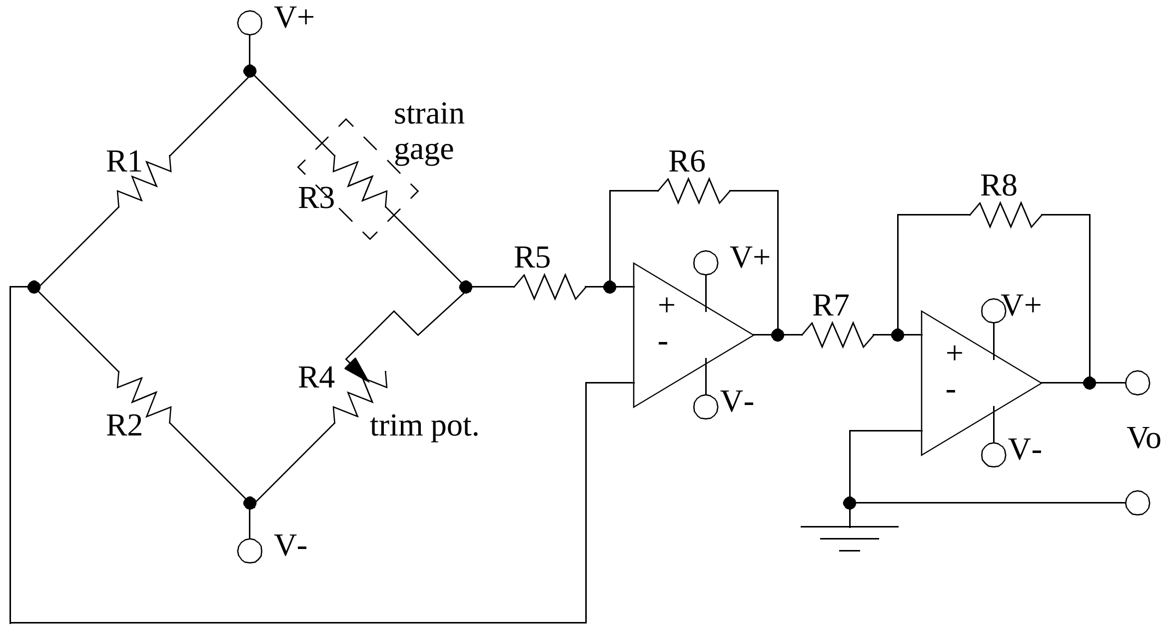


• CONNECTING A SOURCING SENSOR TO A TTL INPUT - This circuit will reduce the larger voltage output from a sourcing sensor (typically 24V) to the lower TTL level (typically 5V).



$$V_i = V_s\left(\frac{R_2}{R_1 + R_2}\right)$$

• STRAIN GAGE AMPLIFIER - this circuit can be used as a crude strain gage amplifier. The ratio of R1/R2 should be close to the ratio of R3/R4. The trim pot can then be used to make minor adjustments. The values of the remaining resistors can be selected to give a suitable amount of isolation.


V+
strain
gage
R1
R3
R6
R8
R5
V+
R7
V+
R4
R2
trim pot.
V-
V-
Vo
V-